YEAR 7 · 7.8 ANGLES

Problem-solving Pack

Name: ______________________________

Date: _______________ Class: __________

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

1 PROBLEM 1 OF 12

Angle chase. In the diagram, three straight lines meet at a point. Two of the six angles formed are labelled: one is 72° and another is 48°. Find all six angles.

(Describe the diagram: two pairs of vertically opposite angles, and a third pair. The 72° and 48° are adjacent to each other.)

Working space

Algebra meets angles. A triangle has angles $x + 10^\circ$, $2x - 5^\circ$, and $3x - 25^\circ$.

Find x, then find each angle. Which type of triangle is it (scalene, isosceles, or equilateral)?

Working space

Regular polygon investigation. A regular polygon has interior angles of 160°.

a) How many sides does it have?
b) What is the sum of all its interior angles?
c) If the polygon is drawn inside a circle, what is the central angle subtended by each side?

Working space

Parallel-line puzzle. Two parallel lines l_1 and l_2 are cut by two transversals, forming a triangle between them.

The two base angles of the triangle (one on each parallel line) are 65° and 50°, measured from the parallel lines. Find the apex angle of the triangle (at the vertex between the parallel lines).

Show the angle reasoning at each step.

Working space

The exterior-angle miracle. Walk around any convex polygon, turning at each vertex. By the time you return to your starting point, you have turned through exactly 360° in total — regardless of the shape or number of sides.

(a) At each vertex of a convex polygon, the exterior angle is the supplement of the interior angle (i.e. exterior = 180° − interior). Show that for a regular hexagon (interior = 120°), each exterior angle is 60°, and that six of these sum to 360°.

(b) For a regular n-gon, write down the exterior angle in terms of n. Use this (not the interior angle formula) to find the exterior angle of a regular 10-gon, 12-gon, and 15-gon.

(c) An irregular convex polygon has exterior angles $52^\circ, 38^\circ, 74^\circ, 62^\circ$, and x°. Find x without knowing the interior angles.

(d) Prove that the exterior angles of **any** convex polygon sum to 360°. Use the fact that the interior angles sum to $(n-2) \times 180^\circ$ and that each exterior angle = $180^\circ -$ interior angle.

Working space

6

Exterior angle theorem. The exterior angle of a triangle is always equal to the sum of the two non-adjacent interior angles.

a) An exterior angle is 110°. One interior angle (non-adjacent) is 47°. Find the other non-adjacent interior angle.
b) Prove the theorem using the fact that angles in a triangle sum to 180° and angles on a straight line sum to 180°.

Working space

Shape and angles. A rhombus has all four sides equal and opposite angles equal. Two of its angles are labelled x and $(2x - 30^\circ)$, where x and $(2x - 30^\circ)$ are opposite angles.

a) Explain why $x = 2x - 30^\circ$ cannot hold, so x and $(2x - 30^\circ)$ must be adjacent angles.
b) Use the fact that adjacent angles in a rhombus are supplementary (sum to 180°) to find x.
c) State all four angles of the rhombus.

Working space

8 PROBLEM 8 OF 12

Interleaved: algebra + angles + factors. The interior angle of a regular polygon is $(180 - \frac{360}{n})^\circ$, where n is the number of sides.

a) Show that for $n = 6$ this gives 120°.
b) Find the smallest value of n for which the interior angle exceeds 170°.
c) Explain why n must always be a factor of 360 for the exterior angle to be a whole number of degrees.

Working space

Reasoning with vertically opposite and parallel angles. In the diagram, two parallel lines are crossed by two transversals that also cross each other between the parallel lines, forming a small triangle.

The angle of the triangle at the left transversal/lower parallel line junction is 55°. The angle at the right transversal/upper parallel line junction is 72°. Find the three angles of the small triangle formed between the two parallel lines.

Show which angle facts you use at each step.

Working space

The star polygon. Draw a regular pentagon (five equal sides and angles). Label the five vertices A, B, C, D, E going clockwise.

Now draw the "star" (pentagram) by connecting every **second** vertex: draw AC, CE, EB, BD, DA. This creates five triangular "points" sticking out of a smaller inner pentagon.

(a) What is the interior angle of a regular pentagon?

(b) Each point of the star is an isosceles triangle. The two base angles of each triangle are angles of the pentagon's diagonals. Using alternate angles or the exterior angle theorem, show that each base angle of a star-point triangle is **72°**.

(c) Find the angle at the **tip** of each star point.

(d) What is the sum of all five tip angles? Does this equal the interior angle sum of a pentagon, a triangle, or neither?

(e) **Extension:** Investigate the six-pointed Star of David (hexagram). What is the angle at each tip? What is the sum of all six tip angles?

Working space

Proof: angles in a polygon. Prove that the sum of interior angles of any n-sided polygon is $(n-2) \times 180^\circ$.

Use the fact that any polygon can be divided into triangles by drawing diagonals from one vertex.

Working space

Interconnected angle puzzle. In a diagram, a straight line AB is drawn. A point C is above the line.
- Angle $CAB = 3x + 5^\circ$
- Angle $CBA = 2x - 10^\circ$
- Angle $ACB = x + 35^\circ$

a) Find x and all three angles.
b) What type of triangle is ABC?
c) The exterior angle at C is drawn. Find it.

$*$(All prior units may be needed: algebra to set up the equation, angle sum rule, triangle classification.)*

Working space