Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. 90° each; 90° + 90° = 180° ✓

2. 115°; check: 65° + 115° = 180° ✓

3. 120°; check: 90° + 80° + 70° + 120° = 360° ✓

4. 50°; check: 70° + 60° + 50° = 180° ✓

5. 85°; check: 90° + 85° + 100° + 85° = 360° ✓

6. 43°, 137°, 43°, 137°; sum = 360° ✓

7. 142°; check: 38° + 142° = 180° ✓

8. 70° each; check: 40° + 70° + 70° = 180° ✓

9. 65°; check: 115° + 65° = 180° ✓

10. Alternate angle = 52°; co-interior angle = 180° − 52° = 128°

SILVER

11. 53°

12. 53°

13. 67°

14. 70°

15. 95°

16. 60°

17. 108°

18. 62°

19. 68°, 112°, 68°, 112°

20. 60°

GOLD

21. x = 50°; largest angle = 100°

22. x = 23.5°; angles are 80.5° and 99.5°

23. 72° and 108°

24. x = 70°; angles are 70°, 90°, 140°, 60°

25. x = 15°; each angle = 48°

26. 150°; dodecagon

27. 15 sides

28. 81°; acute

29. 67°

30. 135°; obtuse

PLATINUM

31. x = 21°; angles are 113° and 67°

32. 90°

33. 12 sides; exterior angle = 30°

34. A = 68°, B = 34°, C = 78°

35. 30°, 60°, 90°

36. 108°

37. Sum = 900°; each angle ≈ 128.57°

38. 57.7°

39. 90°

40. 110° short; reflex angle on the other side = 250°

Pack B — Answers

BRONZE

1. 90° each; 90° + 90° = 180° ✓
2. 68°; check: 112° + 68° = 180° ✓
3. 125°; check: 100° + 75° + 60° + 125° = 360° ✓
4. 55°; check: 90° + 35° + 55° = 180° ✓
5. 85°; check: 110° + 70° + 95° + 85° = 360° ✓
6. 117°, 63°, 117°, 63°; sum = 360° ✓
7. 106°; check: 74° + 106° = 180° ✓
8. 55° each; check: 70° + 55° + 55° = 180° ✓
9. 107°; check: 73° + 107° = 180° ✓
10. Alternate angle = 131°; co-interior angle = 180° − 131° = 49°

SILVER

11. 132°
12. 39°
13. 69°
14. 55°
15. 75°
16. 40°
17. 135°
18. 116°
19. 113°, 67°, 113°, 67°
20. 60°

GOLD

21. x = 52°; largest angle = 104°
22. x = 23.5°; angles are 80.5° and 99.5°
23. 30° and 150°
24. x = 70°; angles are 70°, 90°, 140°, 60°
25. x = 15°; each angle = 48°
26. 144°; decagon
27. 24 sides
28. 48°; acute
29. 67°
30. 150°; obtuse

PLATINUM

31. x = 21°; angles are 113° and 67°
32. 108°
33. 9 sides; exterior angle = 40°
34. A = 66°, B = 33°, C = 81°
35. 20°, 60°, 100°
36. 108°
37. Sum = 1620°; each angle ≈ 147.27°
38. 50.9°
39. 90°
40. 50° short; reflex angle on the other side = 310°

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

72°, 48°, 60°, 72°, 48°, 60°

FULL WORKING

Three straight lines through a single point create six angles that sum to 360°. Opposite angles are equal (vertically opposite), so the angles come in three pairs. Two pairs are known: 72° and 72°, 48° and 48°. The remaining two angles must each be (360° − 2×72° − 2×48°) ÷ 2 = (360° − 144° − 96°) ÷ 2 = 120° ÷ 2 = **60°**. Six angles: 72°, 48°, 60°, 72°, 48°, 60°. Check: 3 × (72 + 48 + 60) = 3 × 180 = 540° — wait, six angles sum to 360°: 2(72) + 2(48) + 2(60) = 144 + 96 + 120 = 360° ✓.

2 Problem 2

ANSWER

x = 33.3̄°; angles are 43.3°, 61.7°, 75°; scalene

FULL WORKING

Sum of angles = 180°: $(x+10)+(2x-5)+(3x-25)=180$. $6x-20=180$. $6x=200$. $x=33.\overline{3}^{\circ}$. Angles: $33.\overline{3}+10=43.\overline{3}^{\circ}$; $2(33.\overline{3})-5=61.\overline{6}^{\circ}$; $3(33.\overline{3})-25=75^{\circ}$. Check: $43.\overline{3}+61.\overline{6}+75=180^{\circ}$ ✓. All three angles differ, so the triangle is **scalene**.

3 Problem 3

ANSWER

a) 18 sides b) 2880° c) 20°

FULL WORKING

a) Exterior angle = 180° − 160° = 20°. Number of sides = 360° ÷ 20° = **18**. **b)** Sum of interior angles = (18 − 2) × 180° = 16 × 180° = **2 880°**. Alternatively, 18 × 160° = 2 880° ✓. **c)** The full 360° is shared equally among 18 sides. Central angle per side = 360° ÷ 18 = **20°** (equal to the exterior angle for a regular polygon inscribed in its circumscribed circle).

4 Problem 4

ANSWER

65°

FULL WORKING

Label the apex angle A and the two base angles $B = 65^\circ$ and $C = 50^\circ$. The angle sum of a triangle is 180°: $A + 65^\circ + 50^\circ = 180^\circ$. $A = 180^\circ - 115^\circ = 65^\circ$.

Alternate-angle confirmation: The 65° base angle is formed between transversal 1 and l_2. The alternate angle (between the same transversal and l_1, inside the triangle) equals 65° (alternate angles, parallel lines). Similarly, the 50° angle on l_1 is an alternate angle inside the triangle. The apex = 180° − 65° − 50° = **65°** ✓.

5 Problem 5

ANSWER

(b) 10-gon: 36°, 12-gon: 30°, 15-gon: 24°. (c) x = 134°. (d) See working.

FULL WORKING

(a) Regular hexagon: interior = 120°, exterior = 180° − 120° = 60°. Sum = 6 × 60° = **360°** ✓.

(b) Exterior angle of regular n-gon = $\frac{360^\circ}{n}$.
- 10-gon: $360^\circ \div 10 = \mathbf{36}^\circ$
- 12-gon: $360^\circ \div 12 = \mathbf{30}^\circ$
- 15-gon: $360^\circ \div 15 = \mathbf{24}^\circ$

(c) Sum of exterior angles = 360°: $52 + 38 + 74 + 62 + x = 360$. $226 + x = 360$. $x = \mathbf{134}^\circ$.

(d) **Proof.** An n-sided convex polygon has n interior angles summing to $(n-2) \times 180^\circ$. Each exterior angle $e_i = 180^\circ - a_i$ where a_i is the corresponding interior angle. Sum of all exterior angles:

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} (180^\circ - a_i) = n \times 180^\circ - \sum_{i=1}^{n} a_i = 180n^\circ - (n-2) \times 180^\circ = 180n^\circ - 180n^\circ + 360^\circ = \mathbf{360}^\circ \ \square$$

This is a beautiful result: no matter how "lumpy" the polygon, the total turning is always exactly one full revolution.

6 Problem 6

ANSWER

a) 63° b) See working.

FULL WORKING

a) By the exterior angle theorem: other angle = 110° − 47° = **63°**. Check: interior angle at the vertex = 180° − 110° = 70°. Triangle sum: 47° + 63° + 70° = 180° ✓.

b) Proof. In triangle ABC, extend side BC to point D. Let the angles of the triangle be $\angle A$, $\angle B$, $\angle C$. The exterior angle is $\angle ACD$. Since BCD is a straight line: $\angle ACB + \angle ACD = 180^\circ$ (angles on a straight line). Since angles in a triangle sum to 180°: $\angle A + \angle B + \angle ACB = 180^\circ$. Subtracting: $\angle ACD = \angle A + \angle B$. The exterior angle equals the sum of the two non-adjacent interior angles. □

7 Problem 7

ANSWER

x = 70°; angles are 70°, 110°, 70°, 110°

FULL WORKING

a) If x and $2x - 30^\circ$ were opposite, they would be equal: $x = 2x - 30$, giving $0 = x - 30$, so $x = 30^\circ$. Then the other angle = $2(30) - 30 = 30^\circ$. Both pairs would be 30°, summing to 120° ≠ 360°. Contradiction — so they must be **adjacent**.

b) Adjacent angles in a rhombus are supplementary: $x + (2x - 30) = 180$. $3x - 30 = 180$. $3x = 210$. $x = 70^\circ$.

c) One pair of opposite angles = 70°. The other pair = $2(70) - 30 = 110^\circ$. Angles: **70°, 110°, 70°, 110°**. Check: $70 + 110 + 70 + 110 = 360^\circ$ ✓.

8 **Problem 8**

ANSWER

a) 120° b) n = 37 c) 360 ÷ n must be an integer

FULL WORKING

a) $180 - \frac{360}{6} = 180 - 60 = 120^\circ$ ✓.

b) We need $180 - \frac{360}{n} > 170$, so $\frac{360}{n} < 10$, so $n > 36$. The smallest whole-number n is **37**.

c) Exterior angle = $\frac{360}{n}$. For this to be a whole number (integer) of degrees, n must divide exactly into 360, i.e. n must be a **factor of 360**. The factors of 360 that are at least 3 (minimum sides for a polygon) are: 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.

9 **Problem 9**

ANSWER

Angles of the triangle: 55°, 72°, 53°

FULL WORKING

Label the lower parallel line l_1 and the upper l_2. The left transversal meets l_1 at angle 55° (inside the triangle). By **alternate angles** (parallel lines), the angle at the top-left vertex of the triangle (where the left transversal meets l_2) equals 55° — but this is an exterior angle; the interior angle is 180° − 55° = 125°...

Re-approach: the 55° is an interior angle of the triangle at the bottom-left vertex. The 72° is an interior angle of the triangle at the top-right vertex. Third angle = 180° − 55° − 72° = **53°**.

Verification with alternate angles: the 55° at l_1 (left transversal) equals the alternate angle on the other side of the left transversal at l_2 — that alternate angle is outside the triangle. The triangle's angles simply sum to 180°: 55° + 72° + 53° = 180° ✓.

10

Problem 10

ANSWER

(a) 108°. (b) Each base angle = 72° (exterior angle of pentagon). (c) Tip angle = 36°. (d) Sum = 5 × 36° = 180° — same as a triangle! (e) Hexagram tip angle = 60°; sum = 6 × 60° = 360°.

FULL WORKING

(a) Interior angle of regular pentagon: exterior angle = $360^\circ \div 5 = 72^\circ$, so interior = $180^\circ - 72^\circ = \mathbf{108^\circ}$.

(b) Consider the star point at vertex A. The two lines meeting at A are the diagonals DA and AC. At vertex D, the interior angle of the pentagon is 108°. The diagonal DA forms part of an isosceles triangle. The angle that diagonal AC makes at vertex C: since BC and CA are two diagonals meeting at C, and the interior angle at C is 108°, the angle $\angle BCA = 108^\circ - \angle DCE$...

Simpler approach: Triangle ABD (the star point at A) has vertices A (the tip), B, and D. Angle ABD is an interior angle of the pentagon = 108°. So angle ABD inside the triangle = 108°... that's too large for a triangle.

Correct approach: The tip triangle at A is $\triangle ACD$... no. The point at A is formed by the intersection of diagonals from other vertices passing near A.

Cleanest approach: Triangle $\triangle ACE$: A, C, E are alternate vertices of the pentagon. This is an isosceles triangle (since $AC = CE$ by symmetry? Not exactly). Let's use the exterior angle theorem:

At point A, two diagonals of the pentagon (DA and CA) form the star's sides. The triangle containing tip A is $\triangle APQ$ where P and Q are the intersection points of the diagonals.

The angle at A in the isosceles triangle: the two sides of the star going through A are parts of the diagonals BD and CE of the pentagon (not DA and CA — the diagonals BD and CE cross near A's region).

Final clean argument: At tip A, the angle equals the exterior angle of the triangle formed by three alternating vertices. In triangle ACE (connecting alternating vertices of the pentagon), the interior angles are all equal (by symmetry) and sum to 180°. So each angle = 60°... but that gives 60°, not 36°.

Correct route using the Exterior Angle Theorem: Consider the full star point triangle at vertex A. It is formed by sides BD and CE (two diagonals of the pentagon) which cross at two points inside the pentagon, and the arc from A to...

Definitive verification: A five-pointed star tip angle = $180^\circ - 2 \times 72^\circ = 180^\circ - 144^\circ = \mathbf{36^\circ}$.

The two base angles of each star-point triangle each equal the exterior angle of the pentagon (72°). This is because each base angle is an alternate angle to an exterior angle of the pentagon formed at an adjacent vertex.

Proof: at the base of the star-point triangle at vertex A, one base angle lies at the crossing of two diagonals inside the pentagon. This angle is vertically opposite to an angle in the interior pentagon, which equals the exterior angle of the original regular pentagon (72°) by the properties of the isosceles triangles formed. $\therefore$ base angles = 72°, tip = $180^\circ - 72^\circ - 72^\circ = \mathbf{36^\circ}$.

(c) Tip angle = **36°**.

(d) Sum of five tip angles = $5 \times 36^\circ = 180^\circ$ — equal to the **interior angle sum of a triangle**!

(e) A regular hexagram (Star of David): formed by two overlapping equilateral triangles. Each star tip is an equilateral triangle's point... actually each tip is a small equilateral triangle (since the hexagon's interior = 120°, exterior = 60°). Tip angle = $180^\circ - 2 \times 60^\circ = \mathbf{60^\circ}$. Sum of six tips = $6 \times 60^\circ = 360^\circ$ — a full revolution.

11 **Problem 11**

ANSWER

$(n-2) \times 180^\circ$

FULL WORKING

Proof. Choose any vertex of an n-sided polygon. Draw diagonals from that vertex to every non-adjacent vertex. This divides the polygon into triangles. Count the triangles: from one vertex, you can draw $(n-3)$ diagonals (to all vertices except the two adjacent ones and itself), creating $(n-2)$ triangles.

Each triangle has an interior angle sum of 180°. The triangles together cover the entire interior of the polygon without overlap. Therefore, the total interior angle sum = $(n-2) \times 180^\circ$. □

Check with examples: Triangle ($n=3$): $(3-2) \times 180^\circ = 180^\circ$ ✓. Square ($n=4$): $2 \times 180^\circ = 360^\circ$ ✓. Pentagon ($n=5$): $3 \times 180^\circ = 540^\circ$ ✓.

12 **Problem 12**

ANSWER

a) x = 25°; angles are 80°, 40°, 60° b) Scalene c) 120°

FULL WORKING

a) Angle sum in triangle $ABC = 180^\circ$: $(3x+5)+(2x-10)+(x+35)=180$. $6x+30=180$. $6x=150$. $x=25$.

Angles: $\angle CAB = 3(25)+5=80^\circ$. $\angle CBA = 2(25)-10=40^\circ$. $\angle ACB = 25+35=60^\circ$.

Check: $80+40+60=180^\circ$ ✓.

b) All three angles differ (80°, 40°, 60°), so all three sides differ in length. Triangle ABC is **scalene**.

c) The exterior angle at C is supplementary to the interior angle at C: $180^\circ - 60^\circ = **120^\circ**$.

By the exterior angle theorem this also equals the sum of the two non-adjacent interior angles: $80^\circ + 40^\circ = 120^\circ$ ✓.