YEAR 7 · 7.4 DECIMALS & MEASUREMENT

Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. E.g. 0.31, 0.35, 0.39 — each is greater than 0.3 and less than 0.4

2. 4.4; the 2nd decimal digit (7) decided — it is 5 or more, so round up

3. Correct answer is 7.22; the student did not align the decimal points correctly

4. 180 mm and 0.18 m; multiplying by 10 converts cm to mm, dividing by 100 converts cm to m

5. 4.85 g

6. Both methods give 7.2

7. Greater than 2 (since 8 ÷ 4 = 2 and 8.4 > 8); exact answer: 2.1

8. 47 mm; multiplied by 10 because 1 cm = 10 mm, so cm → mm requires × 10

9. 3.5 kg; divide by 1 000 because kg is a larger unit (there are 1 000 g in each kg)

10. E.g. 3.2 cm × 1.5 cm (since 2×(3.2+1.5)=9.4) and 4.0 cm × 0.7 cm (since 2×(4.0+0.7)=9.4)

SILVER

11. 5.68

12. 24.40

13. 11.63

14. 8.5

15. 13

16. (a) 235 cm, (b) 2 350 mm

17. 7 cm

18. 12:20 pm

19. 2 450 g

20. Shop B (£1.08 per bottle)

GOLD

21. 6.2

22. Estimate: 4.9 × 3.2 = 15.68; exact: 15.584

23. 3 250 m

24. 8.9 cm

25. 15.05

26. 3.7

27. $x = 1.7$

28. 6 300 s

29. Area = 35.7 cm², Perimeter = 25.4 cm

30. Area = 20.25 cm²; $\sqrt{20.25} = 4.5$ — exact, equals the side length.

PLATINUM

31. $x = 3.2$ cm; area = 22.4 cm²

32. 1.6

33. Rise = 9.2°C; rate = 2.3°C/h

34. $196 = 2^2 \times 7^2$; $\sqrt{196} = 14$

35. $3x + 4.9$; value at $x = 0.5$: 6.4

36. 14 pieces; 10 cm left over

37. 30

38. 5.4

39. 12 minutes

40. Area = 43 200 m² = 4.32 ha

Pack B — Answers

BRONZE

1. E.g. 1.51, 1.55, 1.58 — each is greater than 1.5 and less than 1.6

2. 7.8; the 2nd decimal digit (4) decided — it is less than 5, so round down

3. Correct answer is 10.24; the student did not align the decimal points correctly

4. 245 mm and 0.245 m; multiplying by 10 converts cm to mm, dividing by 100 converts cm to m

5. 4.65 g

6. Both methods give 14.0

7. Greater than 1 (since 5 ÷ 5 = 1 and 7.5 > 5); exact answer: 1.5

8. 38 mm; multiplied by 10 because 1 cm = 10 mm

9. 6.25 kg; divide by 1 000 because kg is a larger unit

10. E.g. 4.6 cm × 2.3 cm (since 2×(4.6+2.3)=13.8) and 5.0 cm × 1.9 cm (since 2×(5.0+1.9)=13.8)

SILVER

11. 3.43

12. 27.03

13. 15.72

14. 14.7

15. 14

16. (a) 408 cm, (b) 4 080 mm

17. 6.4 cm

18. 3:15 pm

19. 3 072 g

20. Shop B (£0.77 per bottle)

GOLD

21. 9.5

22. Estimate: 6.9 × 2.8 = 19.32; exact: 19.432

23. 2 340 m

24. 10.8 cm

25. 11.16

26. −2.8

27. $x = 1.44$

28. 8 400 s

29. Area = 28.88 cm², Perimeter = 22.8 cm

30. Area = 39.69 cm²; $\sqrt{39.69} = 6.3$ — exact, equals the side length.

PLATINUM

31. $x = 3.8$ cm; area = 33.44 cm²

32. 2.4

33. Rise = 9.0°C; rate = 1.8°C/h

34. $225 = 3^2 \times 5^2$; $\sqrt{225} = 15$

35. $4x + 7.2$; value at $x = 1.5$: 13.2

36. 16 pieces; 0 cm left over

37. 20

38. 5.4

39. 10 minutes

40. Area = 28 800 m² = 2.88 ha

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

(a) Yes: 9 − 3 − 1 = 5 (b) Yes: 9 + 3 − 1 = 11 (c) None — you can measure every integer from 1 to 40 (d) 40

FULL WORKING

With weights on both sides, a mass m on the left pan is balanced by some weights on the right pan minus some weights on the left pan (those cancel with m). This is equivalent to writing m as a sum $\sum \epsilon_i w_i$ where $\epsilon_i \in \{-1, 0, +1\}$ (left pan, not used, right pan respectively). This is exactly the balanced-ternary representation!

(a) $5 = 9 - 3 - 1$: put 9 g on the right, and 3 g + 1 g on the same side as the object (left). ✓

(b) $11 = 9 + 3 - 1$: put 9 g and 3 g on the right with the object on the left, and 1 g on the left. ✓

(c) With weights 1, 3, 9, 27 (powers of 3) you can represent every integer from 1 to $1 + 3 + 9 + 27 = 40$ in balanced ternary. There is **no** whole-number mass from 1 to 40 that you cannot measure.

(d) Every integer from 1 to 40 is achievable — that is **40** distinct masses. The key insight: powers of 3 in balanced ternary cover all integers in $[1, (3^n - 1)/2]$ using n weights; here $n = 4$, giving up to $(81 - 1)/2 = 40$.

2 Problem 2

ANSWER

(a) 0.8 m/min; (b) ≈ 1.33 cm/s; (c) ≈ 66.7 mm

FULL WORKING

(a) $0.048\ \text{km/h} = 0.048 \times 1000 = 48\ \text{m/h}$. Per minute: $48 \div 60 = 0.8$ m/min.

(b) $0.8\ \text{m/min} = 80\ \text{cm/min}$. Per second: $80 \div 60 = \frac{4}{3} \approx 1.\overline{3}$ cm/s.

(c) Distance in 5 s: $\frac{4}{3} \times 5 = \frac{20}{3} \approx 6.\overline{6}$ cm $= 66.\overline{6}$ mm ≈ 66.7 mm.

3 Problem 3

ANSWER

$w = 4.7$ cm

FULL WORKING

Total length $= 3.2 + 4.7 + 5.1 = 13.0$ cm.

Perimeter: $2(13.0 + w) = 35.4 \Longrightarrow 13.0 + w = 17.7 \Longrightarrow w = 4.7$ cm.

Check: $2(13.0 + 4.7) = 2 \times 17.7 = 35.4$ ✓.

4 Problem 4

ANSWER

(a) $0.\overline{142857}$ (b) $0.\overline{285714}$ (c) $0.\overline{428571}$ (d) The digits 142857 cycle; see working.

FULL WORKING

(a) Long divide $1.000000 \div 7$: $10 \div 7 = 1$ r 3; $30 \div 7 = 4$ r 2; $20 \div 7 = 2$ r 6; $60 \div 7 = 8$ r 4; $40 \div 7 = 5$ r 5; $50 \div 7 = 7$ r 1 — back to remainder 1. So $\frac{1}{7} = 0.\overline{142857}$.

(b) $\frac{2}{7} = 2 \times \frac{1}{7}$. Doubling each digit of 142857 (carrying as needed): $0.\overline{285714}$.

(c) $\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$.

(d) The six digits 142857 form a cycle. Each fraction $\frac{n}{7}$ simply starts at a different point in the cycle:
- $\frac{4}{7} = 0.\overline{571428}$
- $\frac{5}{7} = 0.\overline{714285}$
- $\frac{6}{7} = 0.\overline{857142}$

Why? Because when you divide by 7, the possible remainders are 1, 2, 3, 4, 5, 6 (in some order). Starting from remainder n gives the decimal for $\frac{n}{7}$, which is just a rotation of the same six-digit block. The number 142857 is called a **cyclic number**.

5 Problem 5

ANSWER

Gamma (£0.43 per 100 ml)

FULL WORKING

Convert all prices to pence per 100 ml for a fair comparison.

- **Alpha**: $240 \div 5 = 48\text{p}$ per 100 ml.
- **Beta**: $345 \div 7.5 = 46\text{p}$ per 100 ml.
- **Gamma**: $1.2\ \text{L} = 1200\ \text{ml}$; $516 \div 12 = 43\text{p}$ per 100 ml.

Gamma is cheapest at 43p per 100 ml $\rightarrow$ **Gamma is best value**.

6 Problem 6

ANSWER

3.57 + 5.46 = 9.03

FULL WORKING

Label the unknowns: $3.A7 + B.4C = 9.03$.

Hundredths column: $7 + C$ must end in 3. So $7 + C = 13 \Longrightarrow C = 6$ (carry 1 to tenths).

Tenths column: $A + 4 + 1\ (\text{carry})$ must end in 0 (tenths digit of 9.03 is 0). So $A + 5 = 10 \Longrightarrow A = 5$ (carry 1 to ones).

Ones column: $3 + B + 1\ (\text{carry}) = 9 \Longrightarrow B = 5$.

The missing digits are $A = 5$, $B = 5$, $C = 6$. Full sum: $3.57 + 5.46 = 9.03$ ✓.

7 Problem 7

ANSWER

(a) 42.2°C; (b) 7 h 42 min; (c) 17 h 35 min

FULL WORKING

(a) Difference $= 23.7 - (-18.5) = 23.7 + 18.5 = 42.2$°C.

(b) Rise needed: $0 - (-18.5) = 18.5$°C. Time $= 18.5 \div 2.4 = 7.708\overline{3}$ h. $0.708\overline{3} \times 60 = 42.5$ min ≈ **7 h 42 min**.

(c) Total rise needed: 42.2°C. Time $= 42.2 \div 2.4 = 17.583\overline{3}$ h. $0.583\overline{3} \times 60 = 35$ min. So **17 h 35 min**.

8 Problem 8

ANSWER

29.2 cm

FULL WORKING

Trace the six sides of the L-shape clockwise from the bottom-left:

1. Bottom: 8.4 cm
2. Right side (lower portion only): $6.2 - 2.5 = 3.7$ cm
3. Notch top (going left): 3.1 cm
4. Notch left edge (going down — this is the inner vertical edge): 2.5 cm
5. Remaining top (going left): $8.4 - 3.1 = 5.3$ cm
6. Left side (full height, going down): 6.2 cm

Perimeter $= 8.4 + 3.7 + 3.1 + 2.5 + 5.3 + 6.2 = 29.2$ cm ✓.

9 Problem 9

ANSWER

(a) 64 (b) 8 (c) 24 (d) 24 (e) 8 (f) 8+24+24+8 = 64 ✓

FULL WORKING

(a) $4 \times 4 \times 4 = \mathbf{64}$ small cubes.

(b) **3 painted faces** — corner cubes. A cube has 8 corners. Each corner small cube touches exactly 3 faces: **8 cubes**.

(c) **2 painted faces** — edge cubes (not corners). Each edge of the large cube has $4 - 2 = 2$ interior positions. A cube has 12 edges, so $12 \times 2 = \mathbf{24}$ cubes.

(d) **1 painted face** — face cubes (not on any edge). Each face of the large cube has a $(4-2) \times (4-2) = 2 \times 2 = 4$ inner grid. 6 faces × 4 = **24** cubes.

(e) **0 painted faces** — interior cubes. These form a $(4-2)^3 = 2^3 = \mathbf{8}$ cube inside.

(f) $8 + 24 + 24 + 8 = 64$ ✓.

Extension for the curious: For an $n \times n \times n$ cube: corners always = 8; edges = $12(n-2)$; faces = $6(n-2)^2$; interior = $(n-2)^3$.

10 Problem 10

ANSWER

(a) 2.35 m ≤ length < 2.45 m; (b) 3 shelves; (c) 2 shelves; (d) result is unreliable — it differs by 1 shelf

FULL WORKING

(a) Rounded to nearest 0.1 m: true length lies in $[2.35, 2.45)$.

(b) Upper bound: $2.45 \div 0.8 = 3.0625 \rightarrow$ **3 complete shelves**.

(c) Lower bound: $2.35 \div 0.8 = 2.9375 \rightarrow$ **2 complete shelves**.

(d) The uncertainty in the measurement means the number of shelves could be either 2 or 3. You cannot rely on the answer without a more precise measurement.

11 Problem 11

ANSWER

£7 623

FULL WORKING

Floor area $= 24.5 \times 18.6$. $24 \times 18.6 = 446.4$; $0.5 \times 18.6 = 9.3$; total $= 455.7$ m².

Each tile covers $0.5 \times 0.5 = 0.25$ m².

Tiles needed (no wastage): $455.7 \div 0.25 = 1\,822.8$ → round up to $1\,823$.

With 10% wastage: $1\,823 \times 1.1 = 2\,005.3$ → round up to $2\,006$ tiles.

Cost: $2\,006 \times £3.80 = 2\,000 \times 3.80 + 6 \times 3.80 = £7\,600 + £22.80 = £7\,622.80 \approx \mathbf{£7\,623}$.

12 Problem 12

ANSWER

(a) Thursday (b) Friday (c) Monday (d) 365 = 52×7 + 1 (advance 1); 366 = 52×7 + 2 (advance 2)

FULL WORKING

(a) From 1 January to 1 February = 31 days. $31 = 4 \times 7 + 3$. So the day advances 3 positions: Mon + 3 = **Thursday**.

(b) From 1 January to 1 March = 31 + 29 = 60 days (2024 is a leap year). $60 = 8 \times 7 + 4$. Day advances 4 from Monday: Mon → Tue → Wed → Thu → **Friday**.

(c) From 1 January to 1 July = Jan(31) + Feb(29) + Mar(31) + Apr(30) + May(31) + Jun(30) = 182 days. $182 = 26 \times 7 + 0$. The day does **not** advance — it is still **Monday**! (This is a surprising result worth noticing: 182 = exactly 26 weeks.)

(d) A normal year has 365 days. $365 = 52 \times 7 + 1$, so the same date advances by 1 day of the week each non-leap year. A leap year has 366 days. $366 = 52 \times 7 + 2$, so it advances by 2 days. This explains why after a leap year, dates "skip" a day of the week.