

# Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

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PROBLEM 1 OF 12

**Diophantine puzzle.** Find all pairs of positive integers  $(x, y)$  that satisfy  $3x + 5y = 47$ .

List every solution. Explain why there are finitely many.

Working space

**Number line puzzle.** Three points A, B, C lie on a number line. A is at  $-14$ . B is exactly halfway between A and C. The distance from A to C is 22.

- (a) What is the coordinate of C?
- (b) What is the coordinate of B?
- (c) What is  $B \times A$ ?

Working space

**Temperature number line investigation.** The temperatures ( $^{\circ}\text{C}$ ) recorded at six cities at midnight are:

Oslo:  $-18$ , Moscow:  $-23$ , Rome:  $4$ , Cairo:  $12$ , Reykjavik:  $-7$ , Seoul:  $-11$ .

- (a) Write the cities in order from coldest to warmest.
- (b) What is the range of temperatures?
- (c) A city X has a temperature exactly halfway between Moscow and Cairo. What is X's temperature?
- (d) If every temperature rises by  $15^{\circ}\text{C}$ , how many cities are then above  $0^{\circ}\text{C}$ ?

Working space

**Magic square with negatives.** Complete the  $3 \times 3$  magic square below so that every row, column, and diagonal has the same sum. Some entries are given.

|                      |                      |                      |
|----------------------|----------------------|----------------------|
| -5                   | <input type="text"/> | 3                    |
| <input type="text"/> | -1                   | <input type="text"/> |
| 1                    | <input type="text"/> | -3                   |

Working space

**Frog jumping puzzle.** Frogs sit on lily pads in a row. Red frogs face right; blue frogs face left. They need to swap sides. The rules:

- A frog can move one space forward onto an empty pad.
- A frog can jump over exactly one frog of the **opposite** colour onto an empty pad.
- No frog may move backwards.

(a) With 1 red and 1 blue frog (3 pads: R \_ B  $\rightarrow$  B \_ R), what is the minimum number of moves?

(b) With 2 red and 2 blue frogs (5 pads), what is the minimum number of moves?

(c) With 3 red and 3 blue frogs (7 pads), what is the minimum number of moves?

(d) Spot the pattern and predict the minimum moves for  $n$  red and  $n$  blue frogs.

Working space

**Sign rules investigation.** Without using a calculator, decide whether each expression is positive or negative. Then calculate the exact value.

(a)  $(-3) \times (-4) \times (-2)$

(b)  $(-2)^5$

(c)  $(-1)^{100}$

(d)  $\frac{(-6)^2}{(-3)}$

Working space

**Factor and sign puzzle.** Two integers satisfy all three conditions:

- Their product is **+36**
- Their sum is **negative**
- Both integers are negative

List all possible pairs. Which pair has the smallest (most negative) sum?

Working space

**Prime factorisation and directed numbers.** The prime factorisation of a number  $n$  is  $2^3 \times 3^2$ .

- (a) Write down the value of  $n$ .
- (b) Write down all the factors of  $n$  that are greater than 10.
- (c) A temperature starts at  $-n^\circ\text{C}$ . It rises by  $\sqrt{n}^\circ\text{C}$ . What is the new temperature?  
(Leave your answer in terms of a square root if  $n$  is not a perfect square.)

Working space

**Directed number patterns.** Here is a pattern of directed numbers:

Row 1:  $-1$

Row 2:  $-2, -1$

Row 3:  $-3, -2, -1$

Row 4:  $-4, -3, -2, -1$

(a) What is the sum of the numbers in Row 4?

(b) Find a formula for the sum of Row  $n$ .

(c) For which row is the sum equal to  $-55$ ?

Working space

**Overdrawn accounts.** Three friends have the following bank balances: Alice £-24, Ben £-15, Chris £48.

- (a) Who has the least money? Who has the most?
- (b) Chris transfers enough money to Alice and Ben so that all three balances are equal. How much does each person end up with?
- (c) How much did Chris transfer in total?

Working space

**Connecting HCF and negative numbers.** Two numbers  $p$  and  $q$  satisfy:

- $p \times q = -180$
- $\text{HCF}(|p|, |q|) = 6$
- $p > 0$  and  $q < 0$
- $|p| < |q|$

Find all possible pairs  $(p, q)$ .

Working space

**Temperature data.** A class records the daily high temperature ( $^{\circ}\text{C}$ ) for one week:

$$-3, 1, -5, 2, -1, 4, -2$$

- (a) What is the range of temperatures?
- (b) What is the mean temperature? (Give your answer as a fraction if not exact.)
- (c) On how many days was the temperature below the mean?
- (d) The following week every temperature is double the previous week's. Does the mean double? Does the range double? Explain.

Working space