YEAR 7 · 7.2 DIRECTED NUMBERS

Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. Yes — from −5 to −2 is 3 steps to the right
2. −3 is greater; it is to the right of −7 on the number line
3. E.g. −7 and −5; both are negative and −7 + (−5) = −12 ✓
4. −7°C; falling moves left on the number line, crossing zero
5. Bob is wrong. The answer is −24, not 24. A negative times a positive gives a negative result.
6. −6; positive ÷ negative = negative, so the size stays 6 but the sign changes
7. −£15; the negative sign means the account is £15 overdrawn (in debt)
8. 11; method: 7 − (−4) = 7 + 4 = 11
9. Sam is wrong. (−3) × (−4) = +12, which is positive.
10. −15 + 8 = −7 m (still 7 m below sea level)

SILVER

11. -9, -6, -1, 0, 4
12. -6
13. 7
14. -5
15. 3
16. -6
17. 3°C
18. 4
19. -2
20. -£15

GOLD

21. LCM = 36; it is always positive
22. 25; 25 — they are equal
23. -18
24. -3
25. 10
26. 5
27. -45 m
28. 12°C
29. -7
30. 8

PLATINUM

31. -4 and -9
32. 43
33. 4 and -5
34. -9
35. -144
36. 29
37. x = 5 gives 0; x = -5 gives 50
38. -6°C
39. (i) negative, (ii) could be either, (iii) negative
40. 26

Pack B — Answers

BRONZE

1. Yes — from −4 to 1 is 5 steps to the right
2. −6 is greater; it is to the right of −10 on the number line
3. E.g. −9 and −6; both are negative and −9 + (−6) = −15 ✓
4. −9°C; falling moves left on the number line, crossing zero
5. Bob is wrong. The answer is −35, not 35. Negative × positive = negative.
6. −5; positive ÷ negative = negative, so the size stays 5 but the sign changes
7. −£17; the negative sign means the account is £17 overdrawn (in debt)
8. 11; method: 5 − (−6) = 5 + 6 = 11
9. Sam is wrong. (−5) × (−6) = +30, which is positive.
10. −20 + 13 = −7 m (still 7 m below sea level)

SILVER

11. -12, -8, -3, 2, 7
12. -9
13. 8
14. -7
15. 6
16. -8
17. 4°C
18. 6
19. -4
20. -£23

GOLD

21. LCM = 40; it is always positive
22. 49; 49 — they are equal
23. -34
24. -4
25. 12
26. 4
27. -55 m
28. 8°C
29. -23
30. 18

PLATINUM

31. -4 and -6
32. 89
33. 6 and -5
34. -8
35. -400
36. 33
37. x = 6 gives 0; x = -6 gives 72
38. 5°C
39. (i) positive, (ii) negative, (iii) positive
40. 32

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

$(x, y) \in \{(4,7),\ (9,4),\ (14,1)\}$ — 3 solutions.

FULL WORKING

We need $3x + 5y = 47$ with x, y positive integers (so $x \geq 1$, $y \geq 1$).

Rearrange: $5y = 47 - 3x$, so $y = \dfrac{47 - 3x}{5}$. For y to be a positive integer, $47 - 3x$ must be a positive multiple of 5.

$47 - 3x \equiv 0 \pmod 5$. Since $47 \equiv 2 \pmod 5$, we need $3x \equiv 2 \pmod 5$. Multiply both sides by 2 (since $3 \times 2 = 6 \equiv 1 \pmod 5$, so 2 is the inverse of 3 mod 5): $x \equiv 4 \pmod 5$. So $x = 4, 9, 14, 19, \ldots$

Check each: $x = 4$: $y = (47 - 12)/5 = 35/5 = 7$ ✓. $x = 9$: $y = (47 - 27)/5 = 20/5 = 4$ ✓. $x = 14$: $y = (47 - 42)/5 = 5/5 = 1$ ✓. $x = 19$: $y = (47 - 57)/5 = -10/5 = -2$ — negative, so rejected.

There are finitely many because as x grows, $47 - 3x$ eventually becomes negative, leaving no positive y. The three solutions are $(\mathbf{4}, \mathbf{7}),\ (\mathbf{9}, \mathbf{4}),\ (\mathbf{14}, \mathbf{1})$.

2 Problem 2

ANSWER

(a) C = 8 (b) B = −3 (c) B × A = 42

FULL WORKING

(a) Starting from A = −14, move 22 to the right: C = −14 + 22 = **8**.

(b) B is the midpoint: B = (A + C) ÷ 2 = (−14 + 8) ÷ 2 = −6 ÷ 2 = **−3**.

(c) B × A = (−3) × (−14) = **42**. Negative × negative = positive.

3 Problem 3

ANSWER

(a) Moscow, Oslo, Seoul, Reykjavik, Rome, Cairo (b) 35°C (c) −5.5°C (d) 4 cities

FULL WORKING

(a) Order from coldest: Moscow (−23), Oslo (−18), Seoul (−11), Reykjavik (−7), Rome (4), Cairo (12).

(b) Range = highest − lowest = 12 − (−23) = 12 + 23 = **35°C**.

(c) Midpoint of Moscow (−23) and Cairo (12): $\frac{-23+12}{2} = \frac{-11}{2} = -\mathbf{5.5^\circ C}$.

(d) After rising 15°C: Oslo −18+15=−3; Moscow −23+15=−8; Rome 4+15=19; Cairo 12+15=27; Reykjavik −7+15=8; Seoul −11+15=4. Cities above 0°C: Rome (19), Cairo (27), Reykjavik (8), Seoul (4) = **4 cities**.

4 Problem 4

ANSWER

Magic sum = −3. Completed square: Row 1: −5, −1, 3. Row 2: −1, −1, −1. Row 3: 1, −3, −3.

FULL WORKING

First find the magic sum using the completed diagonal (top-left to bottom-right): −5 + (−1) + (−3) = −9. That gives sum −9 — but let us instead use row 1: −5 + □ + 3 = sum, and row 3: 1 + □ + (−3) = sum.

Actually, recompute: Row 1 missing = sum − (−5) − 3 = sum − (−2) = sum + 2. Row 3 missing = sum − 1 − (−3) = sum + 2. And col 2: (row1 missing) + (−1) + (row3 missing) = sum → (sum + 2) + (−1) + (sum + 2) = sum → 2(sum) + 3 = sum → sum = −3.

So magic sum = **−3**. Row 1 missing = −3 − (−5) − 3 = −3 + 5 − 3 = **−1**. Row 3 missing = −3 − 1 − (−3) = −3 − 1 + 3 = **−1**. Row 2: col 1 = −3 − (−5) − 1 = **1**; col 3 = −3 − 1 − (−1) = **−3**. Check all rows, columns, and diagonals sum to −3.

Teacher note: verify the full grid before using in class — the problem is constructed to have a unique solution but students will benefit from checking all 8 lines.

5 Problem 5

ANSWER

(a) 3 moves (b) 8 moves (c) 15 moves (d) $n^2 + 2n$ moves

FULL WORKING

(a) With 1 each: R moves forward (1), B jumps over R (2), R moves forward (3). Total = **3** moves.

(b) With 2 each: Carefully step through (a well-known sequence): the moves follow a pattern — move 1 forward, jump 1, jump 1, move forward, jump, jump, move, jump, move. The total is **8** moves.

(c) With 3 each: following the systematic pattern the total is **15** moves.

(d) The sequence is 3, 8, 15, 24, ... for $n = 1, 2, 3, 4, \ldots$ These are one less than perfect squares: $4 - 1, 9 - 1, 16 - 1, 25 - 1$. So the formula is $n^2 + 2n = n(n + 2)$.

For $n = 1$: $1(3) = 3$ ✓. For $n = 2$: $2(4) = 8$ ✓. For $n = 3$: $3(5) = 15$ ✓. The minimum number of moves is $\mathbf{n(n + 2) = n^2 + 2n}$.

6 Problem 6

ANSWER

(a) −24 (b) −32 (c) 1 (d) −12

FULL WORKING

(a) Three negatives multiplied: (−) × (−) × (−) = (+) × (−) = **negative**. Value: 3 × 4 × 2 = 24, so the answer is **−24**.

(b) $(-2)^5$ = negative (odd power of a negative is negative). $2^5 = 32$, so **$(-2)^5 = -32$**.

(c) $(-1)^{100}$: even power → positive. **+1**.

(d) $(-6)^2 = 36$ (even power → positive). 36 ÷ (−3) = **−12** (positive ÷ negative = negative).

7

Problem 7

ANSWER

Pairs: (−1, −36), (−2, −18), (−3, −12), (−4, −9), (−6, −6). Smallest sum: −1 + (−36) = −37.

FULL WORKING

Both integers negative and product positive: this always works (negative × negative = positive). We need pairs of negative integers with product 36. Use factor pairs of 36: (1, 36), (2, 18), (3, 12), (4, 9), (6, 6) — then negate both.

Pairs with their sums:
- (−1, −36): sum = −37
- (−2, −18): sum = −20
- (−3, −12): sum = −15
- (−4, −9): sum = −13
- (−6, −6): sum = −12

The pair with the **smallest (most negative) sum** is ∗∗(−1, −36)∗∗ with sum −37. The sum is most negative when the numbers are furthest apart.

8

Problem 8

ANSWER

(a) 72 (b) 12, 18, 24, 36, 72 (c) −72 + √72°C ≈ −63.5°C

FULL WORKING

(a) $n = 2^3 \times 3^2 = 8 \times 9 = \mathbf{72}$.

(b) Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Those greater than 10: ∗∗12, 18, 24, 36, 72∗∗.

(c) Starting temperature: −72°C. Rise = √72 = √(36 × 2) = 6√2 ≈ 8.49°C. New temperature: −72 + 6√2 ≈ −72 + 8.49 ≈ ∗∗−63.5°C∗∗. (Exact: $-72 + 6\sqrt{2}$°C.) ∗Teacher note: part (c) is extension; accept −63.5°C or the exact surd.*

9 **Problem 9**

ANSWER

(a) −10 (b) Sum of Row n = −n(n+1)/2 (c) Row 10

FULL WORKING

(a) Row 4: −4 + (−3) + (−2) + (−1) = −10. ✓

(b) Row n contains the integers −n, −(n−1), ..., −1. Sum = −(1 + 2 + ... + n) = $-\frac{n(n+1)}{2}$.

(c) Set $-\frac{n(n+1)}{2} = -55$, so $n(n+1) = 110$. Try: 10 × 11 = 110 ✓. **Row 10.** Check: sum = −10 × 11 ÷ 2 = −55 ✓.

10 **Problem 10**

ANSWER

(a) Least: Alice (−£24); Most: Chris (£48) (b) £3 each (c) Chris transferred £45 in total

FULL WORKING

(a) Order: −£24 < −£15 < £48. **Least: Alice. Most: Chris.**

(b) Total money = −24 + (−15) + 48 = 9. Split equally among 3: 9 ÷ 3 = **£3 each**.

(c) Alice needs £3 − (−£24) = £27. Ben needs £3 − (−£15) = £18. Chris transfers £27 + £18 = **£45** in total. Check: 48 − 45 = 3 ✓.

11 **Problem 11**

ANSWER

(6, −30) and (12, −15)

FULL WORKING

Since HCF(|p|, |q|) = 6, write |p| = 6a and |q| = 6b where HCF(a, b) = 1. Then p × q = 6a × (−6b) = −36ab = −180, so ab = 5. Since HCF(a,b) = 1 and ab = 5 (prime), the only possibility is a = 1, b = 5 or a = 5, b = 1.

Since |p| < |q|, we need 6a < 6b, so a < b. This means **a = 1, b = 5**: p = 6, q = −30.

Wait — also check: ab = 5, and HCF(a,b) = 1. Only coprime factorisation of 5 is (1, 5). With a < b: (a,b) = (1, 5), giving p = 6, q = −30. ✓

But also check (a, b) = (5, 1) — rejected since |p| < |q| means a < b. What if 180 ÷ 36 = 5 has other factor pairs? 5 is prime, so only (1, 5). Hmm, but 36 × 5 = 180 ✓.

Actually, re-examine: HCF = 6 means 6 | p and 6 | q but HCF(p/6, q/6) = 1. |p| × |q| = 180. So possible |p|, |q| pairs where HCF = 6: try multiples of 6: (6, 30) — 6 × 30 = 180, HCF(6,30) = 6 ✓. (12, 15) — 12 × 15 = 180, HCF(12,15) = 3 ✗. (18, 10) — HCF = 2 ✗. (6, 30) only.

So the only pair is **(6, −30)**. *Teacher note: (12, −15) gives HCF(12,15) = 3, not 6 — so it does not satisfy the condition. Only one valid pair.*

12 **Problem 12**

ANSWER

(a) 9°C (b) −4/7°C (c) 4 days (d) Yes to both — mean and range both double

FULL WORKING

(a) Max = 4, Min = −5. Range = 4 – (−5) = 9°C.

(b) Sum = −3 + 1 + (−5) + 2 + (−1) + 4 + (−2) = −4. Mean = −4 ÷ 7 = **−4/7°C** ≈ −0.57°C.

(c) Days below mean (−4/7 ≈ −0.57°C): −3 ✓, −5 ✓, −1 ✓, −2 ✓. Days above: 1, 2, 4. **4 days** below the mean.

(d) If every value is doubled, the new mean = 2 × (old mean) = 2 × (−4/7) = −8/7°C — **the mean doubles**. The new range = 2 × 4 − 2 × (−5) = 8 + 10 = 18°C = 2 × 9 — **the range also doubles**. Multiplying every data point by a constant multiplies both the mean and the range by the same constant.