

Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

1

PROBLEM 1 OF 12

Compound percentage. A jacket originally costs £80. It is reduced by 25% in a sale. The following week, the sale price is reduced by a further 10%.

What is the final price? Is the total reduction 35%? Explain why or why not.

Working space

Fraction wall. In a school of 360 students:

- $\frac{3}{8}$ study French.
- $\frac{1}{4}$ of those French students also study Spanish.
- The remaining French students study French only.

How many students study French only?

Working space

Pipe filling. Pipe A fills a tank in 4 hours. Pipe B fills the same tank in 6 hours.

If both pipes are open together, how long does it take to fill the tank?

Working space

Fractions that sum to 1. The ancient Egyptians only wrote fractions with numerator 1 (called "unit fractions"), such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

(a) Write $\frac{5}{6}$ as a sum of **two different** unit fractions.

(b) Write $\frac{7}{12}$ as a sum of **two different** unit fractions.

(c) Show that $\frac{2}{n} = \frac{1}{n} + \frac{1}{n}$ does not count (same fraction twice). Instead, find unit fractions $\frac{1}{a}$ and $\frac{1}{b}$ with $a < b$ such that $\frac{1}{a} + \frac{1}{b} = \frac{2}{9}$.

(d) Is it always possible to write any proper fraction as a sum of unit fractions? Try $\frac{3}{7}$.

Working space

Percentage chain. A shop increases all prices by 20% in January. In July, it reduces all prices by 20%.

A customer claims "the prices are back to where they started." Is she correct? Give a numerical example to support your answer.

Working space

Fraction of the way. A road is 24 km long. A cyclist has completed $\frac{5}{8}$ of the journey.

- (a) How far has she cycled?
- (b) What fraction of the journey remains?
- (c) If the remaining distance takes 45 minutes, what is her average speed in km/h for that section?

Working space

Average with constraints. Five **different** positive integers have mean 6 and median 5.

- (a) What is their sum?
- (b) The smallest is 1. Find all possible sets of five integers satisfying every condition.
- (c) If the smallest must be 2 instead of 1, does a valid set still exist? Explain.

Working space

Simplifying with prime factors.

- (a) Write 126 and 210 each as a product of prime factors.
- (b) Hence find the HCF of 126 and 210.
- (c) Use the HCF to simplify $\frac{126}{210}$ fully.

Working space

Fractions and algebra. The perimeter of a rectangle is 15 cm. One side has length $2\frac{1}{4}$ cm.

Find the length of the other side as a mixed number.

Working space

Farey sequence investigation. A Farey sequence F_n contains all fractions between 0 and 1 (inclusive) with denominators at most n , written in ascending order.

$$F_3: \frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$$

- (a) Write out F_4 in full (all fractions with denominators 1, 2, 3, or 4, in order).
- (b) Pick any two adjacent fractions in F_4 , say $\frac{a}{b}$ and $\frac{c}{d}$. Calculate $bc - ad$. What do you notice?
- (c) For two adjacent Farey fractions $\frac{a}{b}$ and $\frac{c}{d}$, the mediant is $\frac{a+c}{b+d}$. Find the mediant of $\frac{1}{3}$ and $\frac{1}{2}$. Is it between them?
- (d) Verify that the mediant of two adjacent Farey fractions always lies strictly between them.

Working space

Mixed units. A recipe needs $\frac{3}{4}$ kg of flour for 12 biscuits.

- (a) How much flour is needed for 20 biscuits?
- (b) A bag holds 1.5 kg. What fraction of the bag is used for 20 biscuits?
- (c) What percentage of the bag is left over?

Working space

Jamie's monthly budget. Jamie earns £960 per month.

- He saves $\frac{1}{4}$ of his earnings.
- He spends 35% of the remainder on rent.
- He spends $\frac{2}{5}$ of what is left on food and bills.

How much does Jamie have left each month for discretionary spending?

Working space