YEAR 7 · 7.5 FRACTIONS & PERCENTAGES

Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. E.g. $\frac{51}{102}$; equivalent because $\frac{1}{2} = \frac{1\times51}{2\times51} = \frac{51}{102}$

2. $\frac{2}{3}$; HCF = 3; no further simplification possible because 2 and 3 share no common factor other than 1

3. $\frac{5}{8}$; same denominator means same-size pieces, so 5 pieces is more than 3 pieces

4. $\frac{1}{2}$; the denominator stays 4 because the pieces are the same size — you only count up the pieces

5. $\frac{1}{2}$; check: $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ ✓

6. 8; dividing by 3 splits the amount into 3 equal parts, and one part is the answer

7. 40; Method 1: 50% = ½, so 80 ÷ 2 = 40. Method 2: 50% = 50/100, so 80 × 50 ÷ 100 = 40.

8. $\frac{5}{3}$; the whole number 1 contributes $\frac{3}{3}$, plus $\frac{2}{3}$ gives $\frac{5}{3}$

9. $3\frac{1}{2}$; check: $3 \times 2 + 1 = 7$, so $\frac{7}{2}$ ✓

10. 0.5; $\frac{1}{2}$ means 1 divided by 2, and $1 \div 2 = 0.5$

SILVER

11. $\frac{5}{6}$

12. $\frac{5}{12}$

13. $\frac{1}{2}$

14. $1\frac{1}{2}$

15. 36

16. (a) 0.35 (b) $\frac{7}{20}$

17. 16

18. $3\frac{3}{4}$

19. $\frac{3}{5} < \frac{2}{3} < \frac{7}{10}$

20. £48

GOLD

21. HCF = 6; $\frac{3}{4}$

22. −£12

23. 4

24. 2

25. £78

26. $0.\overline{3}$ (recurring)

27. $-\frac{2}{3} < -\frac{1}{2}$

28. $\frac{5}{2}$ (or 2.5)

29. 75 cm

30. 120

PLATINUM

31. £54

32. £60

33. $x = 21$

34. $\frac{17}{16}$

35. $3\frac{13}{24}$

36. $-\frac{3}{4} < -\frac{1}{2} < \frac{1}{3} < \frac{2}{5}$

37. $\frac{1}{3}$

38. 30% increase

39. 16

40. Shop B (27.5p per 100g vs 30p per 100g)

Pack B — Answers

BRONZE

1. E.g. $\frac{34}{51}$; equivalent because $\frac{2}{3} = \frac{2\times17}{3\times17} = \frac{34}{51}$

2. $\frac{2}{3}$; HCF = 4; 2 and 3 share no common factor other than 1

3. $\frac{7}{10}$; same denominator, so 7 pieces out of 10 is more than 4 pieces out of 10

4. $\frac{3}{5}$; the denominator stays 5 because the piece-size (fifths) does not change

5. $\frac{2}{3}$; check: $\frac{2}{3} + \frac{1}{6} = \frac{5}{6}$ ✓

6. 8; dividing by 4 splits the amount into 4 equal parts

7. 30; Method 1: 50% = ½, so 60 ÷ 2 = 30. Method 2: 50% = 50/100, so 60 × 50 ÷ 100 = 30.

8. $\frac{11}{4}$; each whole is $\frac{4}{4}$; two wholes give $\frac{8}{4}$, plus $\frac{3}{4}$ gives $\frac{11}{4}$

9. $2\frac{1}{4}$; check: $2 \times 4 + 1 = 9$, so $\frac{9}{4}$ ✓

10. 0.25; $\frac{1}{4}$ means 1 divided by 4, and $1 \div 4 = 0.25$

SILVER

11. $\frac{7}{12}$

12. $\frac{7}{12}$

13. $\frac{1}{2}$

14. $2\frac{2}{3}$

15. 24

16. (a) 0.60 (b) $\frac{3}{5}$

17. 27

18. $3\frac{1}{2}$

19. $\frac{3}{8} < \frac{5}{12} < \frac{1}{2}$

20. £69

GOLD

21. HCF = 6; $\frac{5}{7}$

22. −£9

23. $2\frac{11}{12}$

24. $2\frac{3}{4}$

25. £110

26. $0.\overline{2}$ (recurring)

27. $-\frac{3}{4} < -\frac{5}{8}$

28. 3

29. 250 cm

30. 180

PLATINUM

31. £81.60

32. £72

33. $x = 24$

34. $\frac{25}{36}$

35. $3\frac{7}{12}$

36. $-\frac{5}{6} < -\frac{2}{3} < \frac{1}{4} < \frac{3}{8}$

37. $\frac{7}{11}$

38. 25% decrease

39. 18

40. Shop B (33p per 100g vs 36p per 100g)

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

£54; total reduction is 32.5%, not 35%.

FULL WORKING

After 25% off: $£80 \times 0.75 = £60$. After a further 10% off: $£60 \times 0.90 = £54$. Total reduction from original: $£80 - £54 = £26$. As a percentage: $\frac{26}{80} \times 100 = 32.5\%$. The reductions are **not** simply added (35%) because the second 10% is taken off the already-reduced price, not the original £80. Sequential percentage changes multiply: $0.75 \times 0.90 = 0.675$, giving a single reduction of $1 - 0.675 = 32.5\%$.

2 Problem 2

ANSWER

101 (or 102 depending on rounding — see working)

FULL WORKING

$\frac{3}{8}$ of 360 = $360 \div 8 \times 3 = 135$ students study French. $\frac{1}{4}$ of 135 = $135 \div 4 = 33.75$. Since we need a whole number of students, round to 34 who also study Spanish. French-only students: $135 - 34 = \mathbf{101}$. Note: the problem's fractions do not combine to give whole numbers with 360 students — a useful teaching point about mathematical modelling. If we round down (33 bilingual), French-only = 102.

3 Problem 3

ANSWER

$2\frac{2}{5}$ hours (2 hours 24 minutes)

FULL WORKING

In one hour, Pipe A fills $\frac{1}{4}$ of the tank and Pipe B fills $\frac{1}{6}$. Together they fill $\frac{1}{4} + \frac{1}{6}$ per hour. LCD = 12: $\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$ per hour. Time to fill whole tank = $1 \div \frac{5}{12} = \frac{12}{5} = 2\frac{2}{5}$ hours = **2 hours 24 minutes**.

4 Problem 4

ANSWER

(a) $\frac{1}{2}+\frac{1}{3}$ (b) $\frac{1}{3}+\frac{1}{4}$ (c) $\frac{1}{5}+\frac{1}{45}$ (d) Yes: $\frac{3}{7}=\frac{1}{3}+\frac{1}{11}+\frac{1}{231}$ (one way)

FULL WORKING

(a) $\frac{5}{6}=\frac{1}{2}+\frac{1}{3}$. Check: $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ ✓.

(b) $\frac{7}{12}=\frac{1}{3}+\frac{1}{4}$. Check: $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$ ✓.

(c) We need $\frac{1}{a}+\frac{1}{b}=\frac{2}{9}$ with $a<b$, $a\neq b$. Since $\frac{1}{a}>\frac{1}{9}$ (as it's the larger part), we need $a<9$. Also $\frac{1}{a}<\frac{2}{9}$ means $a>\frac{9}{2}=4.5$, so $a\geq 5$. Try $a=5$: $\frac{1}{b}=\frac{2}{9}-\frac{1}{5}=\frac{10}{45}-\frac{9}{45}=\frac{1}{45}$. So $b=45$. Check: $\frac{1}{5}+\frac{1}{45}=\frac{9}{45}+\frac{1}{45}=\frac{10}{45}=\frac{2}{9}$ ✓. Answer: $\frac{1}{5}+\frac{1}{45}$.

(d) $\frac{3}{7}$: try $a=3$ (largest unit fraction less than $\frac{3}{7}$ since $\frac{1}{3}\approx 0.333<\frac{3}{7}\approx 0.429...$ actually $\frac{1}{3}<\frac{3}{7}$ since $7<9$). $\frac{3}{7}-\frac{1}{3}=\frac{9}{21}-\frac{7}{21}=\frac{2}{21}$. Now write $\frac{2}{21}$ as a unit fraction sum: $a=11$ gives $\frac{1}{b}=\frac{2}{21}-\frac{1}{11}=\frac{22}{231}-\frac{21}{231}=\frac{1}{231}$. So $\frac{3}{7}=\frac{1}{3}+\frac{1}{11}+\frac{1}{231}$. It is always possible (Fibonacci/Sylvester's sequence guarantees this).

5 Problem 5

ANSWER

She is wrong. The final price is 96% of the original.

FULL WORKING

Take an example: original price £100. After 20% increase: $£100\times 1.20=£120$. After 20% decrease: $£120\times 0.80=£96$. The price is now **£96**, which is **less** than the original £100. Multiplying the multipliers: $1.20\times 0.80=0.96$, a 4% overall **decrease**. The 20% increase and 20% decrease do not cancel because the decrease is calculated on the higher (post-increase) price.

6 Problem 6

ANSWER

(a) 15 km (b) $\frac{3}{8}$ (c) 12 km/h

FULL WORKING

(a) $\frac{5}{8}$ of 24 km: $24\div 8\times 5=15$ km. (b) Remaining fraction: $1-\frac{5}{8}=\frac{3}{8}$. Remaining distance: $\frac{3}{8}\times 24=9$ km. (c) 45 minutes = $\frac{3}{4}$ hour. Speed = distance ÷ time = $9\div\frac{3}{4}=9\times\frac{4}{3}=12$ km/h.

7 Problem 7

ANSWER

(a) 30 (b) Many valid sets, e.g. {1, 2, 5, 8, 14}, {1, 3, 5, 7, 14} — see working for all (c) Yes — e.g. {2, 3, 5, 6, 14}.

FULL WORKING

(a) Mean = 6, five values: sum = $6 \times 5 = \mathbf{30}$.

(b) The median is the 3rd value (when ordered), so the 3rd value = 5. We have: $a < b < 5 < d < e$ with $a + b + 5 + d + e = 30$, so $a + b + d + e = 25$.

Smallest = 1 (given): $a = 1$. So $b + d + e = 24$ with $1 < b < 5$, $5 < d < e$, all different integers.

$b \in \{2, 3, 4\}$.

- $b = 2$: $d + e = 22$, $d \geq 6$. Try $d = 6, e = 16$; $d = 7, e = 15$; $d = 8, e = 14$; $d = 9, e = 13$; $d = 10, e = 12$; $d = 11, e = 11$ (equal — invalid). Valid: $\{1, 2, 5, 6, 16\}$, $\{1, 2, 5, 7, 15\}$, $\{1, 2, 5, 8, 14\}$, $\{1, 2, 5, 9, 13\}$, $\{1, 2, 5, 10, 12\}$.
- $b = 3$: $d + e = 21$, $d \geq 6$. Valid: $\{1, 3, 5, 6, 15\}$, $\{1, 3, 5, 7, 14\}$, $\{1, 3, 5, 8, 13\}$, $\{1, 3, 5, 9, 12\}$, $\{1, 3, 5, 10, 11\}$.
- $b = 4$: $d + e = 20$, $d \geq 6$. Valid: $\{1, 4, 5, 6, 14\}$, $\{1, 4, 5, 7, 13\}$, $\{1, 4, 5, 8, 12\}$, $\{1, 4, 5, 9, 11\}$.

Many valid sets exist (the problem asks to "find all possible sets" — students can list them).

(c) If smallest = 2: $a = 2$, so $b + d + e = 23$, $2 < b < 5$ means $b \in \{3, 4\}$.
- $b = 3$: $d + e = 20$, $d \geq 6$. Gives $\{2, 3, 5, 6, 14\}$ etc. — **valid** ✓.

So a valid set **does** still exist with smallest = 2. (The answer "No" above was incorrect — corrected here: **Yes**, e.g. $\{2, 3, 5, 6, 14\}$.)

8 Problem 8

ANSWER

(a) $126 = 2 \times 3^2 \times 7$; $210 = 2 \times 3 \times 5 \times 7$ (b) HCF = 42 (c) $\frac{3}{5}$

FULL WORKING

(a) $126 \div 2 = 63$; $63 \div 3 = 21$; $21 \div 3 = 7$; 7 prime. $126 = 2 \times 3^2 \times 7$. $210 \div 2 = 105$; $105 \div 3 = 35$; $35 \div 5 = 7$; 7 prime. $210 = 2 \times 3 \times 5 \times 7$. (b) HCF: take lowest powers of shared primes ($2^1, 3^1, 7^1$): $2 \times 3 \times 7 = 42$. (c) $\frac{126}{210} = \frac{126 \div 42}{210 \div 42} = \frac{3}{5}$.

9 Problem 9

ANSWER

$5\,\frac{1}{4}$ cm

FULL WORKING

Perimeter = $2(l+w)=15$, so $l+w=7.5=7\frac{1}{2}$. One side $w=2\frac{1}{4}$. Other side $l=7\frac{1}{2}-2\frac{1}{4}$. Convert: $7\frac{1}{2}=7\frac{2}{4}$. So $l=7\frac{2}{4}-2\frac{1}{4}=5\frac{1}{4}$ cm. Check: $5\frac{1}{4}+2\frac{1}{4}=7\frac{1}{2}$; perimeter = $2\times 7.5=15$ ✓.

10 Problem 10

ANSWER

(a) $\frac{0}{1},\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{1}{1}$ **(b)** $bc-ad=1$ **always (c)** $\frac{2}{5}$**, yes (d) See working.**

FULL WORKING

(a) Fractions with denominator ≤ 4, in order: $\frac{0}{1},\ \frac{1}{4},\ \frac{1}{3},\ \frac{1}{2},\ \frac{2}{3},\ \frac{3}{4},\ \frac{1}{1}$.

(b) Check adjacent pairs: $\frac{0}{1}$ and $\frac{1}{4}$: $1\times 1-0\times 4=1$. $\frac{1}{4}$ and $\frac{1}{3}$: $1\times 4-1\times 3=1$. $\frac{1}{3}$ and $\frac{1}{2}$: $1\times 3-1\times 2=1$. The cross-product $bc-ad=\mathbf{1}$ for every adjacent pair. This is a remarkable property of Farey sequences.

(c) Mediant of $\frac{1}{3}$ and $\frac{1}{2}$: $\frac{1+1}{3+2}=\frac{2}{5}$. Is $\frac{1}{3}<\frac{2}{5}<\frac{1}{2}$? $\frac{1}{3}\approx 0.333$, $\frac{2}{5}=0.4$, $\frac{1}{2}=0.5$. Yes, $\frac{2}{5}$ lies strictly between them.

(d) If $\frac{a}{b}<\frac{c}{d}$ are adjacent Farey fractions, their mediant is $\frac{a+c}{b+d}$. To show $\frac{a}{b}<\frac{a+c}{b+d}$: cross-multiply: $a(b+d)<b(a+c)\Leftrightarrow ab+ad<ab+bc\Leftrightarrow ad<bc\Leftrightarrow bc-ad>0$, which holds since $bc-ad=1>0$. Similarly $\frac{a+c}{b+d}<\frac{c}{d}$ since $d(a+c)<c(b+d)\Leftrightarrow cd-ad<bc+cd-dc\Leftrightarrow$ same condition. So the mediant always lies strictly between adjacent Farey fractions.

11 Problem 11

ANSWER

(a) $1\,\frac{1}{4}$ **kg (b)** $\frac{5}{6}$ **(c)** $16.\overline{6}\%$

FULL WORKING

(a) Flour per biscuit: $\frac{3}{4}\div 12=\frac{3}{48}=\frac{1}{16}$ kg. For 20 biscuits: $\frac{1}{16}\times 20=\frac{20}{16}=\frac{5}{4}=1\frac{1}{4}$ kg. (b) Fraction of bag: $\frac{5/4}{3/2}=\frac{5}{4}\times\frac{2}{3}=\frac{10}{12}=\frac{5}{6}$. (c) Fraction remaining: $1-\frac{5}{6}=\frac{1}{6}$. As a percentage: $\frac{1}{6}\times 100=16.\overline{6}\%$.

12 Problem 12

ANSWER

£280.80

FULL WORKING

Savings: $\frac{1}{4}$ of £960 = £240. Remainder after saving: $£960 - £240 = £720$. Rent: 35% of £720 = $£720 \times 0.35 = £252$. After rent: $£720 - £252 = £468$. Food and bills: $\frac{2}{5}$ of £468 = $£468 \div 5 \times 2 = £93.60 \times 2 = £187.20$. Discretionary: $£468 - £187.20 = \mathbf{£280.80}$.