

# Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

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**1****PROBLEM 1 OF 12**

**Think-of-a-number chain.** I think of a number. I triple it, then add 5. Then I subtract twice the original number. The result is 11.

What is the number? Show your working using algebra.

Working space

**Perimeter puzzle.** A rectangle has length  $(3x + 1)$  cm and width  $(x + 3)$  cm. Its perimeter equals 48 cm.

Find  $x$ , then calculate the area of the rectangle.

Working space

**Age problem.** Priya is  $x$  years old. Her brother is 5 years older. Their mother is three times Priya's age.

The sum of all three ages is 60. How old is each person?

Working space

**Consecutive integer magic.** Alex claims: "Pick any three consecutive integers. Multiply the outer two together, then subtract the square of the middle one. You always get  $-1$ ."

- (a) Test this with the consecutive integers 5, 6, 7.
- (b) Test it with  $-3$ ,  $-2$ ,  $-1$ .
- (c) Prove algebraically that it always works. Let the middle integer be  $n$ .

Working space

**Equation from context.** Three friends share the cost of a meal equally. A service charge of £6 is added to the total before splitting. Each person ends up paying £14.

Write and solve an equation to find the cost of the meal before the service charge.

Working space

**Directed numbers + algebra.** I think of a number. I subtract 8 from it, then multiply the result by 3. The answer is  $-6$ .

Form and solve an equation to find the number.

Working space

**The number machine — fixed points.** A number machine applies the rule: "Multiply by 3, then subtract 10."

- (a) What output does an input of 7 produce?
- (b) What input gives an output of  $-1$ ?
- (c) Find the "fixed point" — the input that gives the same value as the output. Form and solve an equation.
- (d) What happens to inputs above the fixed point after many repeated applications? What about inputs below it?

Working space

**Temperature puzzle.** The temperature at midnight is  $t$  °C. By 6 am it has risen by 7 °C. By noon it has doubled from the 6 am temperature. The noon temperature is 8 °C.

Find  $t$ , the midnight temperature.

Working space

**Pattern and algebra.** A sequence of patterns is made from square tiles:

- Pattern 1: 5 tiles
- Pattern 2: 9 tiles
- Pattern 3: 13 tiles

- (a) Write an expression for the number of tiles in pattern  $n$ .  
(b) Which pattern uses exactly 41 tiles?  
(c) Is there a pattern that uses exactly 100 tiles? Explain.

Working space

**Staircase numbers.** A "staircase number" is any positive integer that can be written as the sum of two or more consecutive positive integers. For example,  $9 = 4 + 5 = 2 + 3 + 4$ .

- (a) Show that 15 is a staircase number in at least **three** different ways.
- (b) Test whether 16 is a staircase number. (Try all possible staircases with 2, 3, 4, or 5 consecutive integers.)
- (c) Show algebraically that the sum of  $k$  consecutive integers starting from  $n$  is  $kn + \frac{k(k-1)}{2}$ .
- (d) Which powers of 2 are staircase numbers? Make a conjecture.

Working space

**Balance puzzle.** On one side of a balance: 3 identical boxes and a 5 kg weight.  
On the other side: 7 identical boxes.

All boxes are the same mass. The scales are balanced.

Find the mass of one box. What total mass is on each side?

Working space

**Extended reasoning.** Jamie says: "If I square any integer and subtract the integer, the result is always even."

- (a) Test Jamie's claim for  $n = -3$ .
- (b) Prove algebraically that  $n^2 - n$  is always even for any integer  $n$ .
- (c) If  $n^2 - n = 30$ , find all integer solutions for  $n$ .

Working space