Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. $b = 3$; because $3 \times 4 = 12$ and $12 + 3 = 15$ ✓
2. $3n$ is correct; in algebra the coefficient is written before the variable
3. E.g. $5x + 3x$ and $4x + 4x$; both simplify to $8x$ ✓
4. E.g. $x + 4 = 12$; solving: $x = 12 - 4 = 8$ ✓
5. $4n = 28$... wait — $n = 7$ chairs per row; equation from context: "total = rows × per row"
6. $3x + 12$; when $x = 2$: bracket gives $3(6) = 18$; expanded gives $6 + 12 = 18$ ✓
7. $x = 6$; check: $2(6) + 5 = 17$ ✓
8. $4n - 3$ and $4 \times n - 3$ are the same; in algebra, $4n$ means $4 \times n$
9. Yes, both have y; $9y - 4y = 5y$ because $9 - 4 = 5$
10. Coefficient of x is 7 (multiplies x); constant is 3 (never changes)

SILVER

11. 18
12. $7x + 2y$
13. $4x + 17$
14. $x = 6$
15. Expression: $5p$; cost = £40
16. 48
17. $8a + 2b$
18. $(2x + 14)$ cm
19. $8x - 20$
20. $x = 15$

GOLD

21. $6x$
22. 8
23. $5x + 16$
24. $x = -2$
25. $3p + 4q$ pence (cannot simplify further as p and q are different)
26. -18
27. $x = 4$
28. $x = 4$
29. $x - 12$
30. 53

PLATINUM

31. $n = 2$
32. 15, 16, 17
33. $x = 4$
34. The difference is always 5.
35. $C = 12 + 3g$; $g = 7$ GB
36. $p^2 - p = p(p - 1)$ — always even.
37. $x = 9,\ y = 5$
38. $x = 5$; sides: 13 cm, 14 cm, 10 cm
39. $c = 15$; $4(x - 2) + 6x = 10x - 8$
40. Input for output 7: $x = 5$. Fixed point: $x = 4$

Pack B — Answers

● **BRONZE**

1. $b = 3$; because $3 \times 6 = 18$ and $18 + 3 = 21$ ✓

2. n^2 is correct; repeated multiplication of the same variable is written using index notation

3. E.g. $7x + 4x$ and $6x + 5x$; both simplify to $11x$ ✓

4. E.g. $3x = 21$; solving: $x = 21 \div 3 = 7$ ✓

5. $5n = 45$... $n = 9$ chairs per row; equation from context: "total = rows × per row"

6. $5x + 10$; when $x = 3$: bracket gives $5(5) = 25$; expanded gives $15 + 10 = 25$ ✓

7. $x = 5$; check: $3(5) + 4 = 19$ ✓

8. $6n - 5$ and $6 \times n - 5$ are the same; $6n$ is the conventional shorthand

9. Yes, both have y; $8y - 3y = 5y$ because $8 - 3 = 5$

10. Coefficient of x is 4 (multiplies x); constant is 9 (never changes)

● **SILVER**

11. 16

12. $10x + 2y$

13. $3x + 22$

14. $x = 5$

15. Expression: $7p$; cost = £42

16. 50

17. $9a + 3b$

18. $(2x + 22)$ cm

19. $12x - 21$

20. $x = 20$

● **GOLD**

21. $7x$

22. 21

23. $7x + 18$

24. $x = -2$

25. $5p + 2q$ pence

26. 23

27. $x = 3$

28. $x = 4$

29. $6x - 8$

30. 32

● **PLATINUM**

31. $n = -2$

32. 20, 21, 22

33. $x = 7$

34. The difference is always 5.

35. $C = 15 + 4g$; $g = 9$ GB

36. $p^2 - p = p(p - 1)$ — always even.

37. $x = 13,\ y = 7$

38. $x = 8$; sides: 26 cm, 13 cm, 7 cm

39. $c = 24$; $3(x - 5) + 7x = 10x - 15$

40. Input for output 6: $x = 4$. Fixed point: $x = \frac{10}{3}$

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

6

FULL WORKING

Let the number be n. Triple it: $3n$. Add 5: $3n+5$. Subtract twice the original: $3n+5-2n=n+5$. Set equal to 11: $n+5=11$. Subtract 5: $n=6$. Check: triple 6 = 18; add 5 = 23; subtract twice 6 = 23 − 12 = 11 ✓.

2 Problem 2

ANSWER

$x=5$; **area = 128 cm²**

FULL WORKING

Perimeter $=2(\text{length}+\text{width})=2((3x+1)+(x+3))=2(4x+4)=8x+8$. Set equal to 48: $8x+8=48 \Rightarrow 8x=40 \Rightarrow x=5$. Length $=3(5)+1=16$ cm; width $=5+3=8$ cm. Check perimeter: $2(16+8)=48$ ✓. Area $=16\times 8=128$ cm².

3 Problem 3

ANSWER

Priya: 11, brother: 16, mother: 33

FULL WORKING

Priya: x. Brother: $x+5$. Mother: $3x$. Sum: $x+(x+5)+3x=5x+5$. Set equal to 60: $5x+5=60 \Rightarrow 5x=55 \Rightarrow x=11$. Priya is 11, her brother is $11+5=16$, her mother is $3\times 11=33$. Check: $11+16+33=60$ ✓.

4 Problem 4

ANSWER

(a) −1 (b) −1 (c) Proof: $(n-1)(n+1)-n^2=n^2-1-n^2=-1$, always.

FULL WORKING

(a) Outer two: 5 and 7. $5 \times 7 = 35$. Middle squared: $6^2 = 36$. $35 - 36 = \mathbf{-1}$ ✓.

(b) Outer two: (-3) and (-1). $(-3) \times (-1) = 3$. Middle squared: $(-2)^2 = 4$. $3 - 4 = \mathbf{-1}$ ✓. (This also interleaves Y7.2 directed-number multiplication.)

(c) Let the three consecutive integers be $n-1$, n, $n+1$. Multiply the outer two: $(n-1)(n+1) = n^2 - 1$ (using the difference-of-two-squares pattern). Subtract the middle squared: $(n^2-1) - n^2 = \mathbf{-1}$. This is independent of n, so the result is always −1.

5 Problem 5

ANSWER

£36

FULL WORKING

Let the meal cost $£m$. Total bill including service charge: $m+6$. Split three ways: $\frac{m+6}{3} = 14$. Multiply both sides by 3: $m + 6 = 42$. Subtract 6: $m = 36$. The meal cost £36. Check: $36 + 6 = 42$; $42 \div 3 = 14$ ✓.

6 Problem 6

ANSWER

6

FULL WORKING

Let the number be n. "Subtract 8": $n-8$. "Multiply by 3": $3(n-8)$. Set equal to -6: $3(n-8) = -6$. Divide both sides by 3: $n - 8 = -2$. Add 8: $n = 6$. Check: $3(6-8) = 3(-2) = -6$ ✓.

7 **Problem 7**

ANSWER

(a) 11 (b) 3 (c) $x = 5$ (d) Inputs above 5 grow without bound; inputs below 5 decrease without bound.

FULL WORKING

(a) $3 \times 7 - 10 = 21 - 10 = \mathbf{11}$.

(b) We need $3x - 10 = -1$. Add 10: $3x = 9$. Divide by 3: $x = \mathbf{3}$. Check: $3(3) - 10 = -1$ ✓.

(c) Fixed point: output = input, so $3x - 10 = x$. Subtract x: $2x - 10 = 0$. Add 10: $2x = 10$. Divide by 2: $x = \mathbf{5}$. Check: $3(5) - 10 = 5$ ✓.

(d) Try $x = 6$ (above 5): $3(6) - 10 = 8$; $3(8) - 10 = 14$; the outputs grow larger. Try $x = 4$ (below 5): $3(4) - 10 = 2$; $3(2) - 10 = -4$; $3(-4) - 10 = -22$; the outputs decrease (and become very negative). The fixed point $x = 5$ is an **unstable** equilibrium — inputs close to it move away from it under repeated application.

8 **Problem 8**

ANSWER

$t = -3$ °C

FULL WORKING

At 6 am: $t + 7$. At noon (doubled from 6 am): $2(t + 7)$. Set equal to 8: $2(t + 7) = 8$. Divide by 2: $t + 7 = 4$. Subtract 7: $t = -3$. The midnight temperature was -3 °C. Check: $-3 + 7 = 4$; $4 \times 2 = 8$ ✓.

9 **Problem 9**

ANSWER

(a) $4n + 1$; (b) pattern 10; (c) No

FULL WORKING

(a) The sequence 5, 9, 13, ... increases by 4 each time. This is linear: $\text{tiles} = 4n + 1$ (check: $n = 1 \Rightarrow 5$ ✓, $n = 2 \Rightarrow 9$ ✓). (b) Set $4n + 1 = 41$: $4n = 40$, $n = 10$ ✓. (c) Set $4n + 1 = 100$: $4n = 99$, $n = 24.75$. Since n must be a whole number, no pattern has exactly 100 tiles.

10 Problem 10

ANSWER

(a) Three ways for 15: see working (b) 16 is not a staircase number (c) See algebraic derivation (d) No power of 2 is a staircase number.

FULL WORKING

(a) $15 = 7 + 8$ (two consecutive). $15 = 4 + 5 + 6$ (three consecutive). $15 = 1 + 2 + 3 + 4 + 5$ (five consecutive). ✓

(b) Two consecutive: $n + (n + 1) = 2n + 1$ — always odd. 16 is even, so no. Three consecutive: $n + (n + 1) + (n + 2) = 3n + 3 = 16 \Rightarrow 3n = 13$ — not an integer. Four consecutive: $4n + 6 = 16 \Rightarrow 4n = 10$ — not an integer. Five consecutive: $5n + 10 = 16 \Rightarrow 5n = 6$ — not an integer. $**$16 is not a staircase number.**

(c) Consecutive integers from n to $n + (k - 1)$: sum = $n + (n + 1) + \cdots + (n + k - 1)$. There are k terms each contributing n, giving kn, plus the extras $0 + 1 + 2 + \cdots + (k - 1) = \dfrac{(k-1)k}{2}$. Total: $kn + \dfrac{k(k-1)}{2}$.

(d) Powers of 2: 1, 2, 4, 8, 16, 32, ... Testing each (as in part b), none can be written as a staircase. The sum $kn + \frac{k(k-1)}{2} = \frac{k(2n+k-1)}{2}$. For a power of 2, both factors k and $(2n + k - 1)$ must be powers of 2 — but they have opposite parity (one odd, one even), so the product can only produce a power of 2 if one of them equals 1, giving trivial cases. $**$Conjecture: no power of 2 is a staircase number.** (This is a famous result in number theory.)

11 Problem 11

ANSWER

Each box: 1.25 kg; total each side: 8.75 kg

FULL WORKING

Let the mass of one box be m kg. Balance equation: $3m + 5 = 7m$. Subtract $3m$: $5 = 4m$. Divide by 4: $m = 1.25$. Total on each side: $7 \times 1.25 = 8.75$ kg. Check left: $3 \times 1.25 + 5 = 3.75 + 5 = 8.75$ ✓.

12

Problem 12

ANSWER

(a) 12 (even); (b) proof below; (c) $n = 6$ and $n = -5$

FULL WORKING

(a) $(-3)^2 - (-3) = 9 + 3 = 12$ ✓ (even — interleaves Y7.2 directed numbers). (b) Factorise: $n^2 - n = n(n-1)$. n and $n-1$ are consecutive integers, so one is always even. The product of an even and any integer is even. Therefore $n^2 - n$ is always even ✓. (c) $n(n-1) = 30$. Look for factor pairs of 30 where one factor is one more than the other: $6 \times 5 = 30$ ✓ (so $n = 6$); $(-5) \times (-6) = 30$ ✓ (so $n = -5$). Check: $6^2 - 6 = 30$ ✓; $(-5)^2 - (-5) = 25 + 5 = 30$ ✓.