

Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

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PROBLEM 1 OF 12

Locker puzzle. 100 lockers are numbered 1 to 100, and they are all closed. 100 students walk past them in order.

- Student 1 opens every locker.
- Student 2 closes every **2nd** locker (lockers 2, 4, 6, ...).
- Student 3 changes the state of every **3rd** locker.
- This continues until student 100.

How many lockers are open at the end? Which lockers are they?

Working space

Coin combinations. Using only 5¢, 10¢, and 20¢ coins, in how many different ways can you make exactly 50¢? (Order does not matter — $\{3 \times 10¢, 4 \times 5¢\}$ is one way.)

Working space

The 1089 trick. Take any 3-digit number where the first digit is at least 2 more than the last digit (e.g. 731).

Step 1: Reverse the digits (137).

Step 2: Subtract the smaller from the larger ($731 - 137 = 594$).

Step 3: Reverse your result (495).

Step 4: Add the result from Step 2 to its reverse ($594 + 495$).

(a) Try the trick with 731. What do you get?

(b) Try it with 852. What do you get?

(c) Does it always give 1089? Explain why by using a general 3-digit number with hundreds digit a , tens digit b , and units digit c where $a > c$.

Working space

Calendar logic. 1st January is a Wednesday. What day of the week is 1st March in the same (non-leap) year?

Show your reasoning clearly.

Working space

Handshake problem. At a party, every person shakes hands exactly once with every other person.

- (a) If there are 5 people, how many handshakes are there in total?
- (b) If there are 10 people, how many handshakes are there?
- (c) Find a formula for the number of handshakes when there are n people.
- (d) At a conference there were 190 handshakes in total. How many people attended?

Working space

Three bells. Three bells toll at the start of school assembly. After that:

- Bell A tolls every 8 minutes.
- Bell B tolls every 12 minutes.
- Bell C tolls every 18 minutes.

- (a) After how many minutes will all three bells first toll together again?
- (b) How many times does Bell A toll in the first 2 hours (not counting the start)?
- (c) Between the start and the first time all three bells toll together, how many times does Bell B toll on its own (not at the same time as any other bell)?

Working space

Unknown digits. In the multiplication below, **A** and **B** represent single digits (0–9):

$$A3 \times B = 161$$

Find the values of A and B. Show your working.

Working space

Sequence puzzle. Here is a sequence: 2, 6, 12, 20, 30, ...

Find the 10th term, and write a rule for the n th term.

Working space

Number theory. Find the smallest three-digit number that is:

- divisible by 7, **and**
- has a digit sum of 9.

Show how you checked your answer.

Working space

Magic square. In a 3×3 magic square, every row, every column, and both main diagonals have the same sum (the "magic sum").

The grid below has three numbers already placed. Find all nine entries.

2		
	5	
		8

Hint: the magic sum can be found from the diagonal containing 2, 5, 8.

Working space

Optimisation. A farmer has exactly 120 m of fencing. He wants to enclose a rectangular field using all of it.

What are the dimensions that give the **largest possible area**? What is that area?

Working space

Modular reasoning. A clock loses exactly 4 minutes every hour.

It is set correctly at **6:00 am**. What time does the clock **show** at **6:00 pm** the same day?

Working space