Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. The missing digit is 4 (tens place of 347)
2. 2 400; the tens digit (6) decided — it is 5 or more, so round up
3. 168; check: 355 + 168 = 523 ✓
4. Closer to 210; 30 × 6 = 180 and 40 × 6 = 240, so 34 × 6 is between, nearer 210
5. Yes — 4728 ÷ 4 = 1182 with no remainder
6. Yes — 84 ÷ 4 = 21 exactly
7. E.g. 4 — because 36 ÷ 4 = 9 (a whole number) ✓
8. 42; because 42 = 7 × 6 and 35 = 7 × 5 which is not greater than 40
9. No — 91 = 7 × 13
10. Less than 100; because 10^2 = 100 and 9 < 10, so $9^2 < 10^2$

SILVER

11. 3 470
12. 4 812
13. 2 375
14. 264
15. 29
16. 12
17. 24
18. 64
19. $2^2 \times 7$
20. £42

GOLD

21. 42
22. 6
23. $2^2 \times 3 \times 7$
24. 6
25. 23 × 24 is larger
26. 48
27. 12 (or 6, 60)
28. 1 000 (estimate)
29. £2.10
30. 15

PLATINUM

31. HCF = 12, LCM = 2 520
32. 5
33. A = 8, B = 4 (since 48 × 3 = 144)
34. (6, 60) and (12, 30)
35. No — 91 = 7 × 13
36. 16 (no — not prime), 25 (no), 34 (no), 43 ✓, 61 ✓, 70 (no). Answer: 43 and 61.
37. a = 8, b = 7
38. Any two consecutive even numbers are 2k and 2(k+1). Their product is 4k(k+1). Since k and k+1 are consecutive integers, one of them is even, so k(k+1) is even. Therefore
39. 3.5

4k(k+1) = 4 × (even) = 8 × (integer). Always divisible by 8.

40. £70

Pack B — Answers

BRONZE

1. The missing digit is 6 (tens place of 463)

2. 8 000; the hundreds digit (8) decided — it is 5 or more, so round up

3. 293; check: 448 + 293 = 741 ✓

4. Closer to 400; 50 × 8 = 400 and 40 × 8 = 320, so 47 × 8 is between, nearer 400

5. Yes — 5913 ÷ 3 = 1971 with no remainder

6. Yes — 135 ÷ 9 = 15 exactly

7. E.g. 6 — because 48 ÷ 6 = 8 (a whole number) ✓

8. 54; because 54 = 9 × 6 and 45 = 9 × 5 which is not greater than 50

9. Yes — 43 has no factors other than 1 and 43

10. Less than 50; because 7^2 = 49 and 8^2 = 64, so 7^2 is between 49 and 64, below 50

SILVER

11. 8 400

12. 8 153

13. 3 247

14. 276

15. 40

16. 6

17. 20

18. 125

19. $2 \times 3 \times 5$

20. £84

GOLD

21. 37

22. 4

23. $2^3 \times 3 \times 5$

24. 7

25. 32 × 33 is larger

26. 180

27. 8 (or 4, 12, 16, 24, 48)

28. 1 800 (estimate)

29. £1.25

30. 13

PLATINUM

31. HCF = 6, LCM = 1 260

32. 13

33. A = 8, B = 4 (same reasoning)

34. (4, 48) and (12, 16)

35. Yes — 97 is prime

36. 17 (1+7=8, prime ✓), 53 (5+3=8, prime ✓), 71 (7+1=8, prime ✓). Answer: 17, 53, 71.

37. a = 12, b = 5

38. Examples: 2×4=8 ✓; 4×6=24=8×3 ✓; 6×8=48=8×6 ✓. General: 2k × 2(k+1) = 4k(k+1). Consecutive integers k, k+1 → one is even → k(k+1) = 2m for some integer m → product = 8m. QED.

39. 3

40. £51.50

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

10 lockers are open: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

FULL WORKING

A locker ends up open if and only if it is toggled an **odd** number of times. Locker n is toggled once for each of its factors. Most numbers have an even number of factors (they pair up: if d divides n, so does $n \div d$). The exception is **perfect squares**, where one factor equals its own pair ($\sqrt{n}$), giving an odd total. The perfect squares from 1 to 100 are $1, 4, 9, 16, 25, 36, 49, 64, 81, 100$ — that is 1^2 through 10^2. So exactly **10 lockers** are open.

2 Problem 2

ANSWER

12 ways

FULL WORKING

Systematically vary the number of 20¢ coins (0, 1, or 2):

- **Two 20¢ coins** (40¢ used, 10¢ remaining): (1×10¢, 0×5¢) or (0×10¢, 2×5¢). **2 ways.**
- **One 20¢ coin** (20¢ used, 30¢ remaining from 5¢ and 10¢): (0×10, 6×5), (1×10, 4×5), (2×10, 2×5), (3×10, 0×5). **4 ways.**
- **No 20¢ coins** (50¢ from 5¢ and 10¢ only): (0×10, 10×5), (1×10, 8×5), (2×10, 6×5), (3×10, 4×5), (4×10, 2×5), (5×10, 0×5). **6 ways.**

Total: 2 + 4 + 6 = **12 ways.** Since the number of 20¢ coins is fixed in each group, the groups are mutually exclusive and exhaustive.

3 Problem 3

ANSWER

(a) 1089 (b) 1089 (c) Always 1089 — see working.

FULL WORKING

(a) 731 reversed = 137. 731 − 137 = 594. 594 reversed = 495. 594 + 495 = **1089** ✓.

(b) 852 reversed = 258. 852 − 258 = 594. 594 reversed = 495. 594 + 495 = **1089** ✓.

(c) Write the number as $100a + 10b + c$. Its reverse is $100c + 10b + a$. Difference (assuming $a > c$): $(100a + 10b + c) - (100c + 10b + a) = 99a - 99c = 99(a - c)$. Since a and c are single digits with $a > c$, we have $a - c \in \{2, 3, 4, 5, 6, 7, 8, 9\}$, giving $99(a - c) \in \{198, 297, 396, 495, 594, 693, 792, 891\}$. These are all 3-digit multiples of 99. For each: its digit-reversal is also in the list, and the sum of any of these with its reversal equals 1089. For example $198 + 891 = 1089$, $297 + 792 = 1089$, $396 + 693 = 1089$, $495 + 495 = 990$ — wait, for $a - c = 5$ we get 495; its reversal 594; $495 + 594 = 1089$ ✓. The result is always **1089**.

4 Problem 4

ANSWER

Saturday

FULL WORKING

January has 31 days; February has 28 days (non-leap year). From 1 January to 1 March is 31 + 28 = 59 days later. 59 ÷ 7 = 8 remainder 3. So 1 March is 3 days after Wednesday: +1 = Thursday, +2 = Friday, +3 = Saturday. **1st March is a Saturday.** (Note: teacher may wish to double-check with a real calendar for a specific year.)

5 Problem 5

ANSWER

(a) 10 (b) 45 (c) $\frac{n(n-1)}{2}$ (d) 20 people

FULL WORKING

(a) Person 1 shakes hands with 4 others, person 2 with 3 remaining others (not counting person 1 again), and so on: $4+3+2+1=\mathbf{10}$ handshakes.

(b) Same pattern: $9+8+7+6+5+4+3+2+1=\mathbf{45}$ handshakes.

(c) Each of the n people shakes hands with $(n-1)$ others. That gives $n(n-1)$ — but each handshake has been counted twice (once for each person). So the formula is $\frac{n(n-1)}{2}$.

(d) Set $\frac{n(n-1)}{2}=190$, so $n(n-1)=380$. We need two consecutive integers with product 380. Try: $19\times 20=380$ ✓. So $n=\mathbf{20}$ people attended.

6 Problem 6

ANSWER

(a) 72 minutes (b) 15 times (c) 2 times

FULL WORKING

(a) We need the LCM of 8, 12, and 18. Prime factorisations: $8=2^3$, $12=2^2\times 3$, $18=2\times 3^2$. LCM = $2^3\times 3^2=8\times 9=\mathbf{72}$ minutes.

(b) In 120 minutes (2 hours), Bell A tolls at: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120. That is $120\div 8=\mathbf{15}$ times.

(c) Bell B tolls at: 12, 24, 36, 48, 60, 72 minutes. We remove times when B coincides with A or C. LCM(8, 12) = 24: B and A both toll at 24, 48, 72. LCM(12, 18) = 36: B and C both toll at 36, 72. Check each: 12 — is it a multiple of 8? $12\div 8=1.5$ ✗; of 18? $12\div 18<1$ ✗ → **B alone** ✓. 24 — multiple of 8 ✓ → with A. 36 — multiple of 18 ✓ → with C. 48 — multiple of 8 ✓ → with A. 60 — $60\div 8=7.5$ ✗, $60\div 18\approx 3.33$ ✗ → **B alone** ✓. 72 — all three. B tolls alone at minutes 12 and 60: **2 times**.

7 Problem 7

ANSWER

A = 2, B = 7 (since 23 × 7 = 161)

FULL WORKING

For $A3 \times B = 161$, the two-digit number $A3$ must end in 3. Possibilities: 13, 23, 33, 43, 53, 63, 73, 83, 93. Divide 161 by each: $161 \div 13 \approx 12.4$ (no); $161 \div 23 = 7$ ✓ (7 is a single digit). Check: $23 \times 7 = 161$ ✓. So $A = 2$ and $B = 7$.

8 Problem 8

ANSWER

110

FULL WORKING

Look at the differences: 6−2=4, 12−6=6, 20−12=8, 30−20=10 — second differences are constant (2), so it is quadratic. Notice: 2 = 1×2, 6 = 2×3, 12 = 3×4, 20 = 4×5, 30 = 5×6. The pattern is $n(n+1)$. For the 10th term: $10 \times 11 = \mathbf{110}$.

9 Problem 9

ANSWER

126

FULL WORKING

Three-digit multiples of 7 start at 105. Check digit sums in order: 105 → 1+0+5=6 (no); 112 → 1+1+2=4 (no); 119 → 1+1+9=11 (no); 126 → 1+2+6=9 ✓. The smallest three-digit multiple of 7 with digit sum 9 is **126**. Check: 126 ÷ 7 = 18 ✓; digit sum = 1+2+6 = 9 ✓.

10 Problem 10

ANSWER

Magic sum = 15. Grid (rows): [2, 7, 6], [9, 5, 1], [4, 3, 8].

FULL WORKING

The main diagonal (top-left to bottom-right) contains 2, 5, 8. These sum to 15, so the **magic sum = 15**.

Now fill the grid. Let the entries be:

$$\begin{array}{|c|c|c|} \hline 2 & a & b \\ \hline c & 5 & d \\ \hline e & f & 8 \\ \hline \end{array}$$

Row 1: $2 + a + b = 15 \Rightarrow a + b = 13.$
Row 3: $e + f + 8 = 15 \Rightarrow e + f = 7.$
Col 1: $2 + c + e = 15 \Rightarrow c + e = 13.$
Col 3: $b + d + 8 = 15 \Rightarrow b + d = 7.$
Anti-diagonal (top-right to bottom-left): $b + 5 + e = 15 \Rightarrow b + e = 10.$

From $a + b = 13$ and $b + d = 7$: subtract $\Rightarrow a - d = 6.$
Col 2: $a + 5 + f = 15 \Rightarrow a + f = 10.$
Row 2: $c + 5 + d = 15 \Rightarrow c + d = 10.$

We know the standard 3×3 magic square using the integers 1–9 has magic sum 15. The only 3×3 magic square (up to rotation/reflection) containing 2, 5, 8 on the leading diagonal uses all of 1–9 exactly once. Testing: rows [2,7,6], [9,5,1], [4,3,8]. Verify: rows: 15,15,15 ✓; cols: 15,15,15 ✓; diagonals: 2+5+8=15, 6+5+4=15 ✓. Answer: **[2,7,6], [9,5,1], [4,3,8]**.

11 Problem 11

ANSWER

30 m × 30 m = 900 m²

FULL WORKING

Let the length be l and the width be w. Perimeter: $2l + 2w = 120$, so $l + w = 60$. Area $A = l \times w$. Try values: if $l = 30, w = 30$: $A = 900$. If $l = 40, w = 20$: $A = 800$. If $l = 50, w = 10$: $A = 500$. The area is largest when the rectangle is a **square** (both sides equal). With $l + w = 60$, the maximum is at $l = w = 30$ giving $A = \mathbf{900}$ m².

12 Problem 12

ANSWER

5:12 pm (the clock shows 11 hours 12 minutes have passed, so 5:12 pm)

FULL WORKING

From 6:00 am to 6:00 pm is 12 real hours. The clock loses 4 minutes every real hour, so in 12 hours it loses $12 \times 4 = 48$ minutes. The clock therefore shows $12 \text{ hours} - 48 \text{ minutes} = 11 \text{ hours and } 12 \text{ minutes}$ elapsed. Starting from 6:00 am + 11 h 12 min = **5:12 pm**. (The clock shows 5:12 pm even though the actual time is 6:00 pm.)