

Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

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PROBLEM 1 OF 12

Fencing a field. A farmer wants to fence a rectangular field with area 72 m^2 . She has exactly 34 m of fencing to use as the perimeter.

Find the length and width of the field. Show all your working.

Working space

Optimal pen design. A farmer has exactly 40 m of fencing. She wants to make a rectangular enclosure divided into **three equal pens** by two internal fences parallel to one pair of sides (as shown below):



Let W be the width of the whole enclosure (perpendicular to the dividers) and L be the length.

- (a) Explain why the total fencing used is $2L + 4W = 40$.
 (b) Express L in terms of W .
 (c) Write the total area A as a function of W alone, and complete the table:

W (m)	1	2	3	4	5	6	7	8	9
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A (m ²)									

- (d) What value of W gives the maximum area? What is that area?
 (e) At the maximum, what is the ratio $L : W$? Does this surprise you?

Working space

Trapezoidal swimming pool. A swimming pool has a trapezoidal cross-section: it is 1 m deep at the shallow end and 3 m deep at the deep end. The pool is 25 m long and 12 m wide.

- (a) Sketch the cross-section and label all dimensions.
- (b) Find the area of the trapezoidal cross-section.
- (c) Find the volume of the pool in m^3 .
- (d) Convert the volume to litres and find how long (in hours) it takes to fill at 1 000 litres per minute.
- (e) A second pool is rectangular with the same length, width, and the same volume of water. How deep is the rectangular pool?

Working space

Tiling a floor. A rectangular kitchen floor is 3.6 m long and 2.4 m wide. Square tiles with side length 30 cm are to be laid with no gaps.

How many tiles are needed? Show how you convert units consistently.

Working space

Shape investigation. A shape is made from a rectangle and a right-angled triangle.

The rectangle is 12 cm long and 5 cm wide. The triangle is attached to one of the shorter ends (5 cm wide), and has a perpendicular height of 8 cm.

- (a) Find the total area of the compound shape.
- (b) Find the perimeter of the compound shape. The slant side of the triangle has length 8.5 cm (given).

Working space

Nets. A cube has side length 4 cm.

- (a) Draw a sketch of a valid net for this cube (describe it in words if you cannot draw).
- (b) Find the total area of the net.
- (c) Explain why a cross-shaped net with five squares in a column and one square to the right of the second square from the top is NOT a valid net for a cube.

Working space

Algebra + area. A rectangle has length $(2x + 4)$ cm and width $(x + 1)$ cm. Its area is 40 cm^2 .

- (a) Show that $x^2 + 3x - 18 = 0$ and solve it to find x .
- (b) Write down the dimensions of the rectangle and find its perimeter.

Working space

Perimeter puzzle. The perimeter of an equilateral triangle equals the perimeter of a square. The square has side length 9 cm.

- (a) Find the side length of the triangle.
- (b) Find the area of the triangle. (Use the formula: $\text{area} = \frac{\sqrt{3}}{4} \times \text{side}^2$, or split into two right-angled triangles.)
- (c) Which shape has the larger area?

Working space

Thinking about 3D shapes. A toy factory uses cuboid boxes with dimensions $6\text{ cm} \times 4\text{ cm} \times 3\text{ cm}$.

- (a) Find the volume and surface area of one box.
- (b) The boxes are packed into a larger cuboid crate. The crate is $24\text{ cm} \times 20\text{ cm} \times 12\text{ cm}$. How many boxes fit in the crate?
- (c) What fraction of the crate's volume is taken up by the boxes? Simplify your answer.

Working space

Area reasoning. Two shapes have the same area.

- Shape A is a triangle with base 16 cm and height h cm.
- Shape B is a trapezium with parallel sides 5 cm and 11 cm, and perpendicular height 8 cm.

Find h .

Working space

Staircase border investigation. A "staircase" pattern is built from unit squares (each 1 cm $\times$ 1 cm). The n -step staircase has n columns: column 1 has 1 square, column 2 has 2 squares, ..., column n has n squares (like a rising staircase from left to right).

A border of width 1 cm is painted around the outside of each staircase.

(a) Draw (or describe) the 1-step, 2-step, and 3-step staircases.

(b) For each of $n = 1, 2, 3$:

- Count the number of unit squares in the staircase.
- Count the perimeter of the staircase (in cm).
- Calculate the area of the 1 cm border painted around it.

(c) Complete the table:

n	Squares in staircase	Perimeter (cm)	Border area (cm ²)
1	1		
2	3		
3	6		
4	10		

(d) Find a formula for the number of unit squares in the n -step staircase.

(e) Find a formula for the border area around the n -step staircase.

(f) The border area of one staircase equals the number of squares in another staircase. Which two values of n satisfy this? (There may be more than one answer.)

Working space

Open-ended investigation. A rectangle has a fixed perimeter of 24 cm.

Complete the table of possible integer dimensions, calculate each area, and identify which dimensions give the maximum area.

Length (cm)	Width (cm)	Area (cm ²)
11	1	
10	2	
9	3	
8	4	
7	5	
6	6	

What do you notice? What happens if the rectangle becomes a square?

Working space