Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. E.g. 6 cm × 2 cm (2×(6+2)=16 ✓) and 7 cm × 1 cm (2×(7+1)=16 ✓)

2. 21 cm; 5 + 7 + 9 = 21

3. E.g. (24, 1), (12, 2), (8, 3) — each verified by multiplication

4. 36 cm²

5. 24 cm²; a triangle is exactly half a rectangle with the same base and height

6. Rectangle (lines through midpoints of opposite sides) and rhombus (lines through opposite vertices)

7. 24 cm³

8. 5 sides (pentagon); 35 ÷ 7 = 5

9. Width = 5 cm; Area = 40 cm²

10. 30 cm²

SILVER

11. 45 cm²

12. 28 cm²

13. 60 cm³

14. 62 cm²

15. 44 cm²

16. 40 cm

17. 6 faces; 52 cm²

18. 7 cm

19. 5 faces, 9 edges, 6 vertices

20. Width = 6 cm; Area = 54 cm²

GOLD

21. 22 cm

22. 2.25 cm² = 225 mm²

23. 5 cm

24. 68 cm²

25. $(6x + 6)$ cm

26. 8 m² = 80 000 cm²

27. 28 cm

28. 122 cm²

29. 45 cm²

30. 120 cm³

PLATINUM

31. x = 5; Area = 33 cm²

32. 72 cm²

33. 180 litres

34. 980 cm² = 0.098 m²

35. 96 cm²

36. New area = 115.2 cm²; 44% increase

37. Height = 5 cm; SA = 148 cm²

38. $196 = 2^2 \times 7^2$; side = 14 cm

39. 80 cm²

40. Side = 6 cm; Volume = 216 cm³ = 0.216 litres

Pack B — Answers

BRONZE

1. E.g. 8 cm × 4 cm (2×(8+4)=24 ✓) and 10 cm × 2 cm (2×(10+2)=24 ✓)
2. 25 cm; 6 + 8 + 11 = 25
3. E.g. (36, 1), (18, 2), (12, 3) — each verified by multiplication
4. 42 cm²
5. 25 cm²; a triangle is half the enclosing rectangle (10 × 5 = 50, halved = 25)
6. Rectangle (lines through midpoints of opposite sides) and rhombus (lines through opposite vertices)
7. 60 cm³
8. 6 sides (hexagon); 48 ÷ 8 = 6
9. Width = 4 cm; Area = 44 cm²
10. 52 cm²

SILVER

11. 84 cm²
12. 40 cm²
13. 96 cm³
14. 108 cm²
15. 61 cm²
16. 42 cm
17. 6 faces; 62 cm²
18. 7 cm
19. 5 faces, 8 edges, 5 vertices
20. Width = 7 cm; Area = 91 cm²

GOLD

21. 28 cm
22. 6.25 cm² = 625 mm²
23. 6 cm
24. 64 cm²
25. $(8x + 10)$ cm
26. 8.1 m² = 81 000 cm²
27. 44 cm
28. 132 cm²
29. 48 cm²
30. 180 cm³

PLATINUM

31. x = 6; Area = 75 cm²
32. 116 cm²
33. 360 litres
34. 1 152 cm² = 0.1152 m²
35. 160 cm²
36. New area = 108.9 cm²; 21% increase
37. Height = 6 cm; SA = 214 cm²
38. $324 = 2^2 \times 3^4$; side = 18 cm
39. 168 cm²
40. Side = 9 cm; Volume = 729 cm³ = 0.729 litres

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

Length = 9 m, width = 8 m (or length = 8 m, width = 9 m)

FULL WORKING

Let the length be l and width w. We have two equations: $lw = 72$ and $2(l + w) = 34$, so $l + w = 17$. This means $w = 17 - l$. Substitute into the area equation: $l(17 - l) = 72$, giving $17l - l^2 = 72$, or $l^2 - 17l + 72 = 0$. Factorise: $(l - 8)(l - 9) = 0$, so $l = 8$ or $l = 9$. If $l = 9$, then $w = 8$. Check: $9 \times 8 = 72$ ✓ and $2(9 + 8) = 34$ ✓. **Length = 9 m, width = 8 m.**

2 Problem 2

ANSWER

(d) W = 5 m gives maximum area 50 m^2. (e) L : W = 10 : 5 = 2 : 1 — the length is always double the width at the optimum.

FULL WORKING

(a) The outer rectangle uses 2 fences of length L (top and bottom) and 2 fences of length W (left and right outer sides), plus 2 internal fences of length W. Total = $2L + 4W = 40$ ✓.

(b) $2L = 40 - 4W$, so $L = 20 - 2W$.

(c) $A = L \times W = (20 - 2W) \times W = 20W - 2W^2$.

W	1	2	3	4	5	6	7	8	9
A	18	32	42	48	**50**	48	42	32	18

Calculations: $A(1) = 20(1) - 2(1) = 18$; $A(2) = 40 - 8 = 32$; $A(3) = 60 - 18 = 42$; $A(4) = 80 - 32 = 48$; $A(5) = 100 - 50 = 50$; $A(6) = 120 - 72 = 48$; etc.

(d) Maximum area = **50 m^2** at $W = 5$ m, $L = 20 - 10 = 10$ m.

(e) $L : W = 10 : 5 = \mathbf{2 : 1}$. This is slightly surprising — the optimal ratio is not a square but 2:1. In general, whenever the interior fences divide the enclosure into k equal pens parallel to a side, the optimal ratio is always $L : W = k : 1$ (here $k = 2$ for the widths, or k internal widths vs 2 external). It shows that the "square is optimal" rule only applies when there are no interior dividers.

3 Problem 3

ANSWER

(b) 50 m² (c) 600 m³ (d) 600 000 litres; 10 hours (e) 2 m deep

FULL WORKING

(a) Cross-section is a trapezium with parallel sides 1 m (top/shallow) and 3 m (bottom/deep), width 25 m along the length of the pool.

(b) Area of trapezium = $\frac{1}{2}(a+b) \times h = \frac{1}{2}(1+3) \times 25 = \frac{1}{2} \times 4 \times 25 = \mathbf{50}$ m². (Here the "height" of the trapezium is the 25 m length of the pool — the cross-section is taken along the length.)

(c) Volume = cross-section area × width = $50 \times 12 = \mathbf{600}$ m³.

(d) $600 \times 1\,000 = \mathbf{600\,000}$ litres. Time = $600\,000 \div 1\,000 = 600$ minutes = **10 hours**.

(e) Rectangular pool: $V = \text{length} \times \text{width} \times \text{depth} = 25 \times 12 \times d = 300d$. Set equal to 600: $300d = 600$, so $d = \mathbf{2}$ m. The trapezoidal pool's average depth is $\frac{1+3}{2} = 2$ m — as expected, since the trapezium equals a rectangle with the average height.

4 Problem 4

ANSWER

96 tiles

FULL WORKING

Convert floor dimensions to cm: 3.6 m = 360 cm; 2.4 m = 240 cm. Number of tiles along length = 360 ÷ 30 = 12. Number of tiles along width = 240 ÷ 30 = 8. Total tiles = 12 × 8 = **96 tiles.** Alternatively: floor area = 3.6 × 2.4 = 8.64 m²; tile area = 0.3 × 0.3 = 0.09 m²; tiles = 8.64 ÷ 0.09 = 96.

5 Problem 5

ANSWER

(a) 80 cm² (b) 45.5 cm

FULL WORKING

(a) Rectangle area = 12 × 5 = 60 cm². Triangle area = ½ × 5 × 8 = 20 cm². Total = 60 + 20 = **80 cm².** **(b)** The outer edges are: top of rectangle (12 cm), far short side (5 cm), slant of triangle (8.5 cm), bottom of rectangle (12 cm), and the left short side (5 cm). But the triangle shares the 5 cm side with the rectangle — that edge is interior. Outer perimeter = 12 (bottom) + 5 (left) + 12 (top) + 8.5 (slant) + 8 (triangle height, right side) = **45.5 cm.**

6 **Problem 6**

ANSWER

(a) Any valid T- or cross-shaped arrangement of six 4 cm × 4 cm squares. (b) 96 cm². (c) That specific arrangement has only 6 squares but when folded, two faces overlap, so it is not a valid net.

FULL WORKING

(a) One valid net: a T-shape — three squares across the top, one square below the middle, two more squares below that, forming a cross. **(b)** A cube has 6 square faces each with area $4^2 = 16$ cm². Total net area = 6 × 16 = **96 cm².** **(c)** When five squares are in a column, folding gives four faces along the "tube" plus top and bottom — however the sixth square position matters: placing it to the right of the second square from the top creates a face that would coincide with another face on folding, so it is invalid. A valid net must allow each of the 6 faces to map to a distinct face of the cube.

7 **Problem 7**

ANSWER

(a) x = 3 (b) Length = 10 cm, width = 4 cm; Perimeter = 28 cm

FULL WORKING

(a) Area = length × width = $(2x+4)(x+1) = 2x^2 + 2x + 4x + 4 = 2x^2 + 6x + 4$. Set equal to 40: $2x^2 + 6x + 4 = 40$. Divide through by 2: $x^2 + 3x + 2 = 20$, so $x^2 + 3x - 18 = 0$. Factorise: $(x+6)(x-3) = 0$, giving $x = -6$ or $x = 3$. Since x must be positive, $x = 3$. **(b)** Length $= 2(3) + 4 = 10$ cm; width $= 3 + 1 = 4$ cm. Check: $10 \times 4 = 40$ ✓. Perimeter $= 2(10+4) = 2 \times 14 = 28$ cm.

8 **Problem 8**

ANSWER

(a) 12 cm (b) 36√3 cm² ≈ 62.4 cm² (c) Square (area = 81 cm²)

FULL WORKING

(a) Perimeter of square = 4 × 9 = 36 cm. So the triangle's perimeter = 36 cm and each side = 36 ÷ 3 = **12 cm.** **(b)** Split the equilateral triangle in half: each half is a right-angled triangle with hypotenuse 12 cm and base 6 cm. Height = $\sqrt{12^2 - 6^2} = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3}$ cm. Area = ½ × 12 × $6\sqrt{3}$ = $36\sqrt{3}$ ≈ 62.4 cm². **(c)** Square area = 81 cm² > 62.4 cm². The **square** has the larger area.

9 **Problem 9**

ANSWER

(a) Volume = 72 cm^3; SA = 108 cm^2 (b) 80 boxes (c) 1 (the boxes fill the crate exactly)

FULL WORKING

(a) Volume = $6 \times 4 \times 3 = 72$ cm^3. SA = $2(6\times4 + 6\times3 + 4\times3) = 2(24 + 18 + 12) = 2 \times 54 =$ **108 cm^2.** **(b)** Boxes along 24 cm: $24 \div 6 = 4$. Along 20 cm: $20 \div 4 = 5$. Along 12 cm: $12 \div 3 = 4$. Total $= 4 \times 5 \times 4 =$ **80 boxes.** **(c)** Crate volume = $24 \times 20 \times 12 = 5\ 760$ cm^3. Total box volume = $80 \times 72 = 5\ 760$ cm^3. Fraction = $5\ 760 \div 5\ 760 =$ **1** — the boxes fill the crate exactly.

10 **Problem 10**

ANSWER

h = 8 cm

FULL WORKING

Area of trapezium (Shape B) = $\frac{1}{2}(5 + 11) \times 8 = \frac{1}{2} \times 16 \times 8 = 64$ cm^2. Set equal to area of triangle: $\frac{1}{2} \times 16 \times h = 64$. So $8h = 64$, h = **8 cm.**

11 Problem 11

ANSWER

(d) Squares = n(n+1)/2. (e) Border area = 4(n+1) cm². (f) 4(n+1) = m(m+1)/2 — e.g. n=1: border=8, squares: m(m+1)/2=8 has no integer solution; n=3: border=16=staircase squares for n=5 (5×6/2=15) — not exact; n=7: border=32=staircase for n=7 (7×8/2=28) — not exact. Closest: border at n=1 is 8 = squares when n=3 gives 6 (not 8); when n=4: 4×5/2=10 (not 8). Actually n=3: border=16; staircase squares = 16 when n(n+1)/2=16 → n²+n-32=0 (not integer). Special case: n=1, border=8; no staircase has exactly 8 squares (closest: n=3→6, n=4→10). This part is open-ended investigation.

FULL WORKING

(a) 1-step: a single 1×1 square. 2-step: column 1 has 2 squares (height 2), column 2 has 1 square (height 1) — an L-shape rotated (or: column 1: 1 square, column 2: 2 squares — a staircase rising right). Using rising-right convention: column k has k squares tall.

(b) and (c):

$n = 1$: 1 square. Perimeter = 4 cm. Border area: the border of width 1 around a shape with perimeter P and convex corners adds $P \times 1 + 4 \times 1^2 = P + 4$ (four quarter-circles at corners become one full square of area 1 each). Border area = $4 + 4 = \mathbf{8}$ cm².

$n = 2$: Squares = $1 + 2 = 3$. Perimeter: trace the outline — left side (2 units up), bottom (2 right), right side (1 down), step left (1 left), step down (1 down), top-left (1 left) = total $2 + 2 + 1 + 1 + 1 + 1 = 8$ cm. Border area = $8 + 4 = \mathbf{12}$ cm².

$n = 3$: Squares = $1 + 2 + 3 = 6$. Perimeter: left (3 up), bottom (3 right), right (1 down), step (1+1), right (1 down), step (1+1), top (1 left) = $3 + 3 + 1 + 1 + 1 + 1 + 1 + 1 = \mathbf{12}$ cm. Border area = $12 + 4 = \mathbf{16}$ cm².

$n = 4$: Squares = 10. Perimeter = $4 \times 4 = 16$ cm (pattern: perimeter = $4n$). Border area = $16 + 4 = \mathbf{20}$ cm².

n	Squares	Perimeter	Border area
1	1	4	8
2	3	8	12
3	6	12	16
4	10	16	20

(d) Squares $= 1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$.

(e) Perimeter $= 4n$ (each step adds 2 horizontal and 2 vertical edges — two edges are "new" outer boundary per step plus the staircase grows linearly). Border area $= 4n + 4 = 4(n+1)$.

(f) We want $4(n+1) = \dfrac{m(m+1)}{2}$, i.e. $8(n+1) = m(m+1)$. Try values: $n = 1$: $8(2) = 16$, $m(m+1) = 16 \rightarrow$ no integer m. $n = 3$: $8(4) = 32$, $m(m+1) = 32 \rightarrow$ no integer. $n = 4$: $8(5) = 40$, $m(m+1) = 40 \rightarrow m = 5$: $5 \times 6 = 30$ (no); $m = 6$: 42 (no). $n = 9$: $8(10) = 80$, $m(m+1) = 80 \rightarrow m = 8$: 72 (no), $m = 9$: 90 (no). $n = 2$:

$8(3) = 24 \to m(m+1) = 24$: no integer. $n = 5$: $8(6) = 48 \to m = 6$: 42 (no), $m = 7$: 56 (no). This is an open investigation — students discover there may be no whole-number solution, which itself is a meaningful finding.

12 Problem 12

ANSWER

Maximum area = 36 cm² when the rectangle is a square (6 cm × 6 cm).

FULL WORKING

Fill the table: 11×1=11; 10×2=20; 9×3=27; 8×4=32; 7×5=35; 6×6=36. Areas increase as dimensions become more equal. The maximum area occurs when the rectangle is a **square** (6 cm × 6 cm), giving 36 cm². This is a specific instance of the general result: for a fixed perimeter, the square maximises area. Students should notice that the areas increase and then reach a peak — a pattern that connects to optimisation.