

Problem-solving Pack

These problems are designed to challenge you. Read each question carefully. Show all your reasoning — a correct answer without working receives no credit.

1**PROBLEM 1 OF 12**

Constructing a data set from constraints. A teacher gives six students a test. You know:

- The mean score is 14.
- The median score is 13.
- The range is 12.
- The mode is 13.
- All scores are whole numbers between 1 and 20 inclusive.

(a) Show that the total of all six scores is 84.

(b) The scores in order are: a , b , 13, 13, d , e . Find all possible sets of scores that satisfy every condition. Explain your reasoning step by step.

Working space

The shifting median. A class of 15 students takes a maths test. Their scores are listed in order, and the median is 72.

- (a) What position is the median in a list of 15 values?
- (b) Five new students join the class. All five score 85. The teacher lists all 20 scores in order and finds the new median. Will the new median definitely be higher than 72, or could it stay at 72, or could it go either way? Explain with a specific example.
- (c) Now suppose instead those five new students all scored 65. What happens to the median? Again, could it stay at 72?
- (d) Find the **minimum** number of new students (all scoring 85) you would need to add to a class of 15 whose median is 72 to **guarantee** the new median is above 72. Justify your answer.

Working space

Pie chart sense-check. A class of 36 students was asked to name their favourite season. The pie chart shows: Spring = 60° , Summer = 150° , Autumn = 90° , Winter = 60° .

- (a) What fraction of the class chose Summer?
- (b) How many students chose Autumn?
- (c) A student says "More than half the class chose Summer." Is she correct? Explain.

Working space

Comparing two classes. Class A quiz scores: 55, 62, 68, 71, 74, 74, 80, 82.
Class B quiz scores: 40, 55, 70, 72, 74, 76, 88, 93.

For each class calculate:

(a) the mean (b) the median (c) the range.

Then write two sentences comparing the two classes.

Working space

Find the data set (reasoning). Five whole numbers satisfy all of these conditions:

- Mean = 8
- Median = 7
- Mode = 7
- Range = 9

Find a possible set of five numbers. Show how you verified each condition.

Working space

Pie chart construction. A survey asked 120 people how they prefer to watch films:

Method	Number of people
Cinema	30
Streaming	54
DVD	18
TV broadcast	18

- (a) Calculate the pie chart angle for each method.
- (b) What percentage prefer Streaming?
- (c) If the pie chart is drawn, what does the Cinema sector look like compared to the Streaming sector?

Working space

Misleading mean. An estate agent says: "The mean house price on Maple Street is £240 000." There are 5 houses on the street. Four sell for £180 000 each, and one sells for £480 000.

- (a) Verify the estate agent's claim by calculating the mean.
- (b) Explain why the mean might be misleading.
- (c) Calculate the median house price. Which average better represents the street?

Working space

The heights puzzle. Five friends measure their heights. You are told:

- The mean height is 160 cm.
- The range of heights is 20 cm.
- Exactly three of the friends are 160 cm tall.

- (a) Show that the sum of all five heights is 800 cm.
- (b) Let the shortest friend have height s cm and the tallest have height t cm. Write two equations connecting s and t .
- (c) Solve to find s and t . Are there other possible solutions?
- (d) **Extension:** If instead two of the five friends are 160 cm tall (not three), and the mean and range are the same, find all possible values of s and t . How many solutions are there now?

Working space

Adjusting data. A teacher records the marks of 8 students: 56, 63, 70, 48, 72, 65, 59, 67.

- (a) Find the current mean.
- (b) The teacher adds 5 bonus marks to every student's score. Without listing all new scores, find the new mean. Explain your reasoning.
- (c) Find the range before and after the bonus marks are added.

Working space

The moving mean mystery. A student takes maths tests one at a time and tracks her running mean after each test.

- After Test 1: mean = 80
- After Test 2: mean = 75
- After Test 3: mean = 80
- After Test 4: mean = 78

- (a) Find the score she got on each of the four tests. Show your method.
- (b) She wants her mean to be exactly 80 after Test 5. What must she score?
- (c) **Investigate:** After n tests the mean is M_n . After test $n + 1$ the mean becomes M_{n+1} . Write a formula for the $(n + 1)$ th test score in terms of n , M_n , and M_{n+1} .
- (d) Use your formula to explain: if a student's mean goes **up** after a new test, what does that tell you about the score she just got?

Working space

Dual statistics. A scientist records temperatures at two weather stations over 5 days.

Station A ($^{\circ}\text{C}$): 4.2, -1.5, 3.8, -0.6, 2.1

Station B ($^{\circ}\text{C}$): 6.5, -3.0, 5.2, -1.8, 3.1

- (a) Find the mean temperature at each station.
- (b) Find the range at each station.
- (c) Station B has a higher mean. Does that mean Station B is warmer overall? Explain using the range.

Working space

Design your own data. Create a data set of exactly **6 values** satisfying ALL of the following:

- The mean is 12.5.
- The median is 11.5.
- The mode is 10.
- The range is 15.
- All values are positive whole numbers.

Verify every condition and explain the reasoning you used to construct the set.

Working space