YEAR 7 · 7.6 STATISTICS

Solutions · Full Answer Key

Pack A answers · Pack B answers · Problem-solving worked solutions

Pack A — Answers

BRONZE

1. Mode = 3; it appears 3 times, which is more than any other value (7 appears twice, 9 and 5 appear once)

2. Median = 5; must sort first (1, 2, 5, 6, 8) so that the middle value represents the centre of the data

3. 4; because mean = total ÷ count, so total = 6 × 3 = 18; third number = 18 − 4 − 10 = 4

4. Range = 15; this means the data spans 15 units — the values are fairly spread out

5. No — double 9 is 18, and 14 < 18, so Fiction is not more than double Non-fiction

6. (a) 28 students; (b) categorical — colours are labels, not numbers

7. Modal size = 5; it has frequency 8, higher than any other size — mode means most common

8. Median = 6; with an even count there is no single middle value, so we average the two middle values to find the centre

9. $\frac{1}{3}$ walk; fractions: $\frac{10}{30} + \frac{15}{30} + \frac{5}{30} = \frac{30}{30} = 1$ ✓

10. Steps of 3 are poor because the highest value (8) does not fall on a gridline; steps of 2 are better (gridlines at 2, 4, 6, 8 — all bars land exactly on gridlines)

SILVER

11. 7.6

12. 11

13. Class A: mean = 7, range = 4. Class B: mean = 6, range = 9. Class A has a higher mean and a smaller range, so it performed better and more consistently.

14. 120°

15. 27

16. 12

17. 12

18. (a) Mode = 4; (b) Median = 5.5

19. 11.5 cm

20. 50

GOLD

21. 85

22. (a) $\frac{1}{4}$ (b) 25%

23. Team A: mean = 12.0 s, range = 1.8 s. Team B: mean = 12.0 s, range = 3.6 s. Both teams have the same mean speed but Team A is more consistent (smaller range).

24. 10

25. (a) Mean = 10.75, Median = 7, Mode = 7. (b) Median or mode — the score of 40 is an outlier that distorts the mean.

26. A = 120°, B = 150°, C = 90°

27. 10

28. 21

29. (a) Mean = £36 000. (b) The mean is pulled up by the two high manager salaries; 8 out of 10 employees earn only £20 000. The median (£20 000) is more representative.

30. One possible set: 3, 6, 6, 9, 11 — but verify against constraints. A correct set: 3, 6, 6, 8, 12 — nope. Valid example: 3, 6, 6, 10, 10 gives mean=(3+6+6+10+10)÷5=35÷5=7, mode=6 (appears twice, but so does 10 — bimodal). Better: 3, 6, 6, 8, 12: range=9 (no). Try: 3, 6, 6, 7, 13: range=10 (no). Try: 4, 6, 6, 8, 11: sum=35, mean=7 ✓, range=7 (no). Try: 3, 6, 6, 9, 11: sum=35 ✓, mean=7 ✓, range=8 ✓, mode=6 ✓. Answer: 3, 6, 6, 9, 11.

PLATINUM

31. One valid set: 8, 9, 10, 13, 15 — check: sum=55, mean=11 ✓; median=10 ✓; range=15−8=7 (no). Try: 8, 9, 10, 12, 16: sum=55 ✓, range=8 (no). Try: 9, 10, 10, 12, 14: sum=55 ✓, range=14−9=5 (no). Try: 8, 10, 10, 13, 14: sum=55 ✓, range=6 ✓, median=10 ✓. Answer: 8, 10, 10, 13, 14.

32. Mean = $\frac{6}{7}$ ≈ 0.9 °C (or exact: $\frac{6}{7}$ °C). Range = 8 °C.

33. £12 profit

34. She must score a total of 160 in the remaining 2 assessments (a mean of 80 per assessment).

35. Archer A: mean = 4 cm, range = 3 cm. Archer B: mean = 5 cm, range = 8 cm. Choose Archer A — lower mean (more accurate on average) and lower range (more consistent).

36. Football: 144°, 40%. Tennis: 72°, 20%. Swimming: 90°, 25%. Other: 54°, 15%.

37. Mean = 13 (a whole number)

38. (a) Teaching: £120 000, Resources: £40 000, Premises: £30 000, Other: £10 000. (b) Premises = 54°.

39. 4.3 cm

40. $x = 9$

Pack B — Answers

BRONZE

1. Mode = 4; it appears 3 times, which is more than any other value (6 appears twice, 8 and 2 appear once)
2. Median = 5; must sort first (1, 3, 5, 7, 9) so the middle value is meaningful
3. 8; because total = 6 × 3 = 18; third number = 18 − 3 − 7 = 8
4. Range = 18; the data spans 18 units between the lowest (2) and highest (20) values
5. No — double 9 is 18, and 20 > 18 but barely; Fiction is more than double since 20 > 18 ✓ — actually yes for Pack B
6. (a) 30 students; (b) categorical — colours are labels/categories, not numerical values
7. Modal size = 6; it has frequency 11, the highest of all sizes
8. Median = 7; with 6 values there is no single middle, so we average the 3rd and 4th values
9. $\frac{1}{2}$ walk; fractions: $\frac{20}{40} + \frac{12}{40} + \frac{8}{40} = \frac{40}{40} = 1$ ✓
10. Steps of 3 are poor because 15 is not a multiple of 3 neatly fitting the chart; steps of 5 are better (gridlines at 5, 10, 15 — all bars land on gridlines)

SILVER

11. 6.2
12. 14
13. Class A: mean = 6, range = 4. Class B: mean = 6, range = 7. The means are equal but Class A has a smaller range, so it is more consistent.
14. 135°
15. 23
16. 12
17. 12
18. (a) Mode = 5; (b) Median = 5.5
19. 10.4 cm
20. 50

GOLD

21. 76
22. (a) $\frac{1}{5}$ (b) 20%
23. Team A: mean = 10.0 s, range = 0.8 s. Team B: mean = 10.0 s, range = 2.8 s. Equal means; Team A is more consistent (smaller range).
24. 11
25. (a) Mean = 6.875, Median = 4, Mode = 4. (b) Median or mode — the score of 30 is an outlier that distorts the mean.
26. A = 135°, B = 90°, C = 135°
27. 16
28. 36
29. (a) Mean = £33 750. (b) The two managers earning £90 000 each inflate the mean well above what most staff earn (£15 000). The median (£15 000) better represents a typical salary.

30. One valid set: 1, 4, 4, 7, 14: sum=30, mean=6 ✓, range=13 (no). Try: 1, 4, 4, 9, 12: sum=30 ✓, range=11 (no). Try: 2, 4, 4, 8, 12: sum=30 ✓, range=10 ✓, mode=4 ✓. Answer: 2, 4, 4, 8, 12.

PLATINUM

31. One valid set: 4, 7, 8, 10, 16 — sum=45, mean=9 ✓; median=8 ✓; range=12 (no). Try: 5, 7, 8, 10, 15: sum=45 ✓, range=10 ✓, median=8 ✓. Answer: 5, 7, 8, 10, 15.

32. Mean = $-\frac{2}{7}$ ≈ −0.3 °C. Range = 11 °C.

33. £10 profit

34. She must score a total of 220 in the remaining 3 assessments (a mean of approximately 73.3 per assessment).

35. Archer A: mean = 5 cm, range = 3 cm. Archer B: mean = 5 cm, range = 9 cm. Same mean; choose Archer A for consistency (smaller range).

36. Football: 144°, 40%. Tennis: 72°, 20%. Swimming: 108°, 30%. Other: 36°, 10%.

37. Mean = 15 (a whole number)

38. (a) Teaching: £75 000, Resources: £37 500, Premises: £22 500, Other: £15 000. (b) Premises = 54°.

39. 6.2 cm

40. $x = 5$

Problem-solving — Worked Solutions

1 Problem 1

ANSWER

Two valid sets: {8, 11, 13, 13, 19, 20} and {8, 12, 13, 13, 18, 20}. See working for full derivation.

FULL WORKING

(a) Mean = 14, six scores: total = $14 \times 6 = \mathbf{84}$.

(b) Ordered scores: $a < b < 13 = 13 < d < e$ (with 13 appearing at least twice for the mode, and median = mean of 3rd and 4th = $\frac{13+13}{2} = 13$ ✓). Range = $e - a = 12$.

So $a + b + 13 + 13 + d + e = 84 \Rightarrow a + b + d + e = 58$. Also $e = a + 12$.

Substituting: $a + b + d + (a + 12) = 58 \Rightarrow 2a + b + d = 46$.

Constraints: $a \geq 1$, $b > a$, $b < 13$, $d > 13$, $e = a + 12 \leq 20$ so $a \leq 8$.

Try $a = 8$: $e = 20$. $b + d = 30 - 2(8) = 14$... wait: $2(8) + b + d = 46 \Rightarrow b + d = 30$. With $b \in \{9, 10, 11, 12\}$ and $d > 13$: $b = 12, d = 18$: valid (9, 12, 13, 13, 18, 20: $b = 12 < 13$ ✓, $d = 18 > 13$ ✓, $e = 20 \leq 20$ ✓). $b = 11, d = 19$: (9,11,13,13,19,20... wait $a = 8$ here). Let me redo: $a = 8, e = 20$: need $b + d = 30$, $8 < b < 13$, $d > 13$. $b = 12, d = 18$: set $\{8, 12, 13, 13, 18, 20\}$; sum$= 84$ ✓, range$= 12$ ✓. $b = 11, d = 19$: $\{8, 11, 13, 13, 19, 20\}$; sum$= 84$ ✓. $b = 10, d = 20$: $d = 20 = e$ — would require two 20s but $d < e$: invalid.

Try $a = 7$: $e = 19$. $b + d = 32$. $b < 13, d > 13$: $b = 12, d = 20$: $\{7, 12, 13, 13, 20, 19\}$ — order problem ($d < e$: $20 > 19$): invalid. $b = 13$: b must be < 13: invalid. So $b \leq 12, d \leq 19$ (since $e = 19$): $b = 12, d = 20$ invalid; $b = 12, d = 19$ but $d < e = 19$: invalid. Hmm, so $d < e = 19$, meaning $d \leq 18$: $b + d \leq 12 + 18 = 30 < 32$: no solution for $a = 7$.

So the valid sets are those with $a = 8$: $\{8, 12, 13, 13, 18, 20\}$ and $\{8, 11, 13, 13, 19, 20\}$. Both have sum 84 ✓, median 13 ✓, mode 13 ✓, range 12 ✓.

2 Problem 2

ANSWER

(a) 8th position. (b) New median is definitely higher than 72 — it sits at position 10.5 (mean of 10th and 11th), both of which are ≥ 72, and with 5 students above the old median the 10th and 11th values are both above 72. (c) New median could stay at 72 or drop below it. (d) Adding 8 students scoring 85 gives 23 values; median is the 12th — guaranteed above original 8th. The answer is 8.

FULL WORKING

(a) In an ordered list of 15 values, the median is the $\frac{15+1}{2} = 8$th value.

(b) After adding 5 students scoring 85, there are 20 values. The new median = mean of the 10th and 11th values. The original 8th value was 72. Adding five 85s pushes all original values 8th–15th down to positions (roughly) 8th–15th, but the five 85s appear after all original values ≤ 85. Key insight: originally, at least 8 values are ≤ 72 (the 8th value and all before it). After adding five 85s, those ≤72 values occupy positions 1–8 (at most). So the 10th and 11th values are both ≥ 72, and since the five new students scored strictly above 72, both positions are at least 72. If the 9th and 10th original values were 72, the new median would be higher. In fact the 10th and 11th positions in the combined list are among the original values from position 8 onwards or the new 85s — all ≥ 72, and with 5 extras at 85 the 10th and 11th are guaranteed to be above the 8th original (72). The new median is **definitely ≥ 72**, and in most realistic cases **strictly above** 72.

Simpler argument: originally 7 values are below the median. After adding five 85s, there are still just 7 values below 72. The new median (10th value of 20) is at least the 10th smallest overall — but 7 values are below 72 and 5 are above, so the 10th value is one of the original values from position 3 onwards, which are ≥ the original 8th (72). The new median is ≥ 72 and likely higher if any of positions 9–11 in the original list exceeded 72.

(c) Adding five students scoring 65: now there are 20 values, and 5 new values below the old median (72). Originally 7 values were below 72; now up to 12 values could be below 72 (7 original + 5 new). The 10th and 11th values in the new ordered list might both be below 72, so the median could **drop below** 72 or stay at 72 depending on the original data.

Example where it drops: original = 60,62,64,66,68,70,72,72,74,76,78,80,82,84,86. Median = 8th = 72. Add five 65s: new ordered list includes 60,62,64,65,65,65,65,65,66,68,70,72,72,72,... The 10th value is 68 < 72. New median = (68+70)÷2 = 69 < 72.

(d) We need the median position of $(15+k)$ values to fall above position 8 in the original list. The new median position is $\frac{15+k+1}{2}$ for odd totals (or mean of two central values for even totals). For the median to be **guaranteed** above 72, position $\lceil \frac{15+k}{2} \rceil > 8$, i.e. $15+k > 16$, so $k > 1$. But we also need the actual value there to exceed 72.

With k new values of 85: they all sit above the original values ≤ 72. The new 8th position is the same original value only if $k \leq 7$ (since original positions 1–8 are still in the first 8 spots of the combined list). When $k = 8$: total = 23 values; median = 12th value. The original list has 8 values at or below 72 in positions 1–8. Adding eight 85s shifts these: positions 1–8 remain the 8 original values ≤ 72, position 9 onward includes original values ≥ 72 and the eight 85s. The 12th value > original 8th (72). So $k = 8$ is needed to guarantee the new median is strictly above 72.

3 Problem 3

ANSWER

(a) 5/12 (b) 9 students (c) No — Summer is 150° out of 360°, which is less than half (180°).

FULL WORKING

(a) $\frac{150}{360} = \frac{5}{12}$ of the class chose Summer.

(b) Autumn sector = $\frac{90}{360} = \frac{1}{4}$. Number of students = $\frac{1}{4} \times 36 = \mathbf{9}$.

(c) Half the circle = 180°. Summer = 150° < 180°, so Summer represents less than half. The student is **incorrect**.

4 Problem 4

ANSWER

Class A: mean = 70.75, median = 72.5, range = 27. Class B: mean = 71, median = 73, range = 53. Class B has a slightly higher mean and median but a much larger range, so Class A is more consistent.

FULL WORKING

∗∗Class A∗∗ (8 values, already ordered): sum = 55+62+68+71+74+74+80+82 = 566; mean = 566÷8 = ∗∗70.75∗∗. Median = mean of 4th and 5th = (71+74)÷2 = ∗∗72.5∗∗. Range = 82−55 = ∗∗27∗∗.

∗∗Class B∗∗: sum = 40+55+70+72+74+76+88+93 = 568; mean = 568÷8 = ∗∗71∗∗. Median = (72+74)÷2 = ∗∗73∗∗. Range = 93−40 = ∗∗53∗∗.

Class B's mean and median are slightly higher, showing marginally better scores on average. However, Class A's range (27) is much smaller than Class B's (53), so Class A performed more consistently.

5 **Problem 5**

ANSWER

One valid set: 4, 7, 7, 9, 13.

FULL WORKING

Total = 8 × 5 = 40. The middle (3rd) value must be 7 (median). At least two values must equal 7 (mode). Choose two 7s as 2nd and 3rd values, with minimum value m and maximum $m+9$ (range = 9).

Values so far (ordered): m, 7, 7, ?, $m+9$.

Remaining 5th value contributes to the total: $m+7+7+?+(m+9)=40 \Rightarrow 2m+?+23=40 \Rightarrow ?=17-2m$.

For the ordering to hold: $7 \leq ? \leq m+9$. Try $m=4$: $?=17-8=9$; check $7 \leq 9 \leq 13$ ✓.

Set: 4, 7, 7, 9, 13. Verify: sum = 40 ✓, mean = 8 ✓, median = 7 ✓, mode = 7 ✓, range = 13−4 = 9 ✓.

6 **Problem 6**

ANSWER

(a) Cinema 90°, Streaming 162°, DVD 54°, TV 54°. (b) 45%. (c) Cinema (90°) is a right-angle sector; Streaming (162°) is nearly half the pie — considerably larger.

FULL WORKING

Multiplier = 360 ÷ 120 = 3°.

(a) Cinema: 30 × 3 = **90°**. Streaming: 54 × 3 = **162°**. DVD: 18 × 3 = **54°**. TV: 18 × 3 = **54°**. Check: 90+162+54+54 = 360° ✓.

(b) Streaming % = $\frac{54}{120} \times 100 = \mathbf{45\%}$.

(c) The Cinema sector is a quarter of the circle (90°), while Streaming is almost half (162°), so Streaming appears nearly twice as large as Cinema.

7 Problem 7

ANSWER

(a) Mean = £240 000 ✓ (b) One very high price pulls the mean up. (c) Median = £180 000; the median better represents the typical price.

FULL WORKING

(a) Sum = 4 × £180 000 + £480 000 = £720 000 + £480 000 = £1 200 000. Mean = £1 200 000 ÷ 5 = **£240 000** ✓.

(b) Four out of five houses cost only £180 000. The one house at £480 000 is an outlier that raises the mean above what most buyers would actually pay. The mean is misleading as a "typical" price.

(c) Ordered prices: £180 000, £180 000, **£180 000**, £180 000, £480 000. Median = 3rd value = **£180 000**. The median better represents the typical house price on this street.

8

Problem 8

ANSWER

(a) 5 × 160 = 800. (b) s + t = 320 and t − s = 20. (c) s = 150 cm, t = 170 cm — the solution is unique. (d) With two at 160 cm: s + t = 480 − 2×160... wait: two at 160 means the remaining two (s and t) must sum to 800 − 3×160 = 320 with t − s = 20, giving the same unique answer s = 150, t = 170. With only two at 160: s + (remaining) + 160 + 160 + t = 800, so s + remaining + t = 480; many solutions exist depending on the 3rd value.

FULL WORKING

(a) Mean = 160, five friends: total = $160 \times 5 = \mathbf{800}$ cm.

(b) Three friends are each 160 cm, contributing $3 \times 160 = 480$ cm. The remaining two friends (shortest s and tallest t) must satisfy:

$$s + t = 800 - 480 = 320$$

The range is $t - s = 20$.

(c) Adding the two equations: $2t = 340$, so $t = \mathbf{170}$ cm. Subtracting: $2s = 300$, so $s = \mathbf{150}$ cm.

Check: $150 + 160 + 160 + 160 + 170 = 800$ ✓. Range $= 170 - 150 = 20$ ✓. Mean $= 800 \div 5 = 160$ ✓.

This solution is **unique** — the system of two equations with two unknowns has exactly one solution. There are no other possibilities.

(d) If exactly **two** of the five friends are 160 cm tall, let the three others have heights s, m, t where $s < m < t$. We need:

$$s + m + t + 160 + 160 = 800 \Rightarrow s + m + t = 480$$

Range $= t - s = 20$, so $t = s + 20$. Substituting: $s + m + (s + 20) = 480 \Rightarrow 2s + m = 460$.

For valid whole-number solutions we need $s < 160 < t$ (otherwise one of the 160 cm people isn't actually the mode), $m \neq 160$, and $m > s$. Since $m = 460 - 2s$, and $s < 160$, $t = s + 20 < 180$: if $s = 150, t = 170, m = 460 - 300 = 160$ — but $m = 160$ gives three people at 160 again (contradiction). So $m \neq 160$: $460 - 2s \neq 160 \Rightarrow s \neq 150$. Any other value of $s < 150$ or $150 < s < 160$ gives a valid m. There are **infinitely many solutions** — adding a third free variable removes the uniqueness.

9 Problem 9

ANSWER

(a) Mean = 62.5 (b) New mean = 67.5 (each score increases by 5, so the mean increases by 5) (c) Range stays the same at 24 both times.

FULL WORKING

(a) Sum = 56+63+70+48+72+65+59+67 = 500. Mean = 500 ÷ 8 = **62.5**.

(b) Adding 5 to every score increases the total by 5 × 8 = 40. New mean = (500+40) ÷ 8 = 540 ÷ 8 = **67.5**. Alternatively: if every value increases by 5, the mean also increases by 5.

(c) Before: Max = 72, Min = 48. Range = 72 − 48 = **24**. After adding 5: Max = 77, Min = 53. Range = 77 − 53 = **24**. The range is unchanged — adding the same constant to every value shifts all scores equally, so the spread remains the same.

10 **Problem 10**

ANSWER

(a) Test 1 = 80, Test 2 = 70, Test 3 = 90, Test 4 = 72. (b) She must score 88. (c) Score = (n+1)·M_{n+1} − n·M_n. (d) If the mean goes up, the new score is above the previous mean.

FULL WORKING

(a) After each test, the total = (number of tests) × (mean at that point).

- After Test 1: total = $1 \times 80 = 80$. Score 1 = **80**.
- After Test 2: total = $2 \times 75 = 150$. Score 2 = $150 - 80 = \mathbf{70}$.
- After Test 3: total = $3 \times 80 = 240$. Score 3 = $240 - 150 = \mathbf{90}$.
- After Test 4: total = $4 \times 78 = 312$. Score 4 = $312 - 240 = \mathbf{72}$.

Scores: 80, 70, 90, 72.

(b) Target after Test 5: mean = 80, so total = $5 \times 80 = 400$. Current total after 4 tests = 312. Score 5 = $400 - 312 = \mathbf{88}$.

(c) After n tests: total = $n \cdot M_n$. After $n+1$ tests: total = $(n+1) \cdot M_{n+1}$. The new score is the difference:

$$\text{Score}_{n+1} = (n+1) \cdot M_{n+1} - n \cdot M_n$$

Check with (a): Score 2 = $2 \times 75 - 1 \times 80 = 150 - 80 = 70$ ✓. Score 3 = $3 \times 80 - 2 \times 75 = 240 - 150 = 90$ ✓.

(d) If $M_{n+1} > M_n$ (the mean went up), then:

$$\text{Score}_{n+1} = (n+1)M_{n+1} - nM_n > (n+1)M_n - nM_n = M_n$$

So the new score is **strictly greater than the previous mean**. A rising mean tells you the latest score was above the old mean — which makes sense intuitively: dragging the average up requires a score above it.

11 Problem 11

ANSWER

(a) Station A mean = 1.6 °C; Station B mean = 2.0 °C. (b) Station A range = 5.7 °C; Station B range = 9.5 °C. (c) Station B has a higher mean but also a much larger range — it experiences bigger swings between warm and cold days, so it is less stable.

FULL WORKING

(a) Station A: sum = 4.2 + (−1.5) + 3.8 + (−0.6) + 2.1 = 8.0. Mean = 8.0 ÷ 5 = **1.6 °C**.
Station B: sum = 6.5 + (−3.0) + 5.2 + (−1.8) + 3.1 = 10.0. Mean = 10.0 ÷ 5 = **2.0 °C**.

(b) Station A: Max = 4.2, Min = −1.5. Range = 4.2 − (−1.5) = **5.7 °C**.
Station B: Max = 6.5, Min = −3.0. Range = 6.5 − (−3.0) = **9.5 °C**.

(c) Although Station B has a higher mean (warmer on average), its range is 9.5 °C compared to 5.7 °C. This means Station B experiences much greater temperature swings — it can be colder or hotter than Station A on any given day. The range shows Station A is more stable.

12 Problem 12

ANSWER

One valid set: 5, 10, 10, 13, 15, 20.

FULL WORKING

Total needed = 12.5 × 6 = 75. The median of 6 values is the mean of the 3rd and 4th values; we need (3rd + 4th) ÷ 2 = 11.5, so 3rd + 4th = 23. Mode = 10, so 10 appears at least twice — place it in positions 1 and 2 (or 2 and 3). Range = 15, so max − min = 15.

Try: positions 1–2 = 10, 10 (mode satisfied). Then 3rd + 4th = 23. Choose 3rd = 10 would make mode 10 appear three times (still fine); 4th = 13. Or 3rd = 11, 4th = 12. Let's keep 3rd = 11, 4th = 12. Min so far = 10. For range = 15, max = min + 15. If min = 5 (position 1 is 5, not 10), we need 10 to appear twice elsewhere.

Revised: positions ordered = 5, 10, 10, 13, ?, 20. Range = 20 − 5 = 15 ✓. Median = (10+13)÷2 = 11.5 ✓. Mode = 10 ✓. Sum = 5+10+10+13+?+20 = 58+?. We need total = 75, so ? = 17. But then we need 5th value ≤ 6th: 17 ≤ 20 ✓, and 4th ≤ 5th: 13 ≤ 17 ✓.

Final set: 5, 10, 10, 13, 17, 20.

Verify: Sum = 5+10+10+13+17+20 = 75 ✓. Mean = 75÷6 = 12.5 ✓. Median = (10+13)÷2 = 11.5 ✓. Mode = 10 (appears twice, most frequent) ✓. Range = 20−5 = 15 ✓.